Methods and applications for signomial programming in engineering design

Berk Ozturk
06/08/2018

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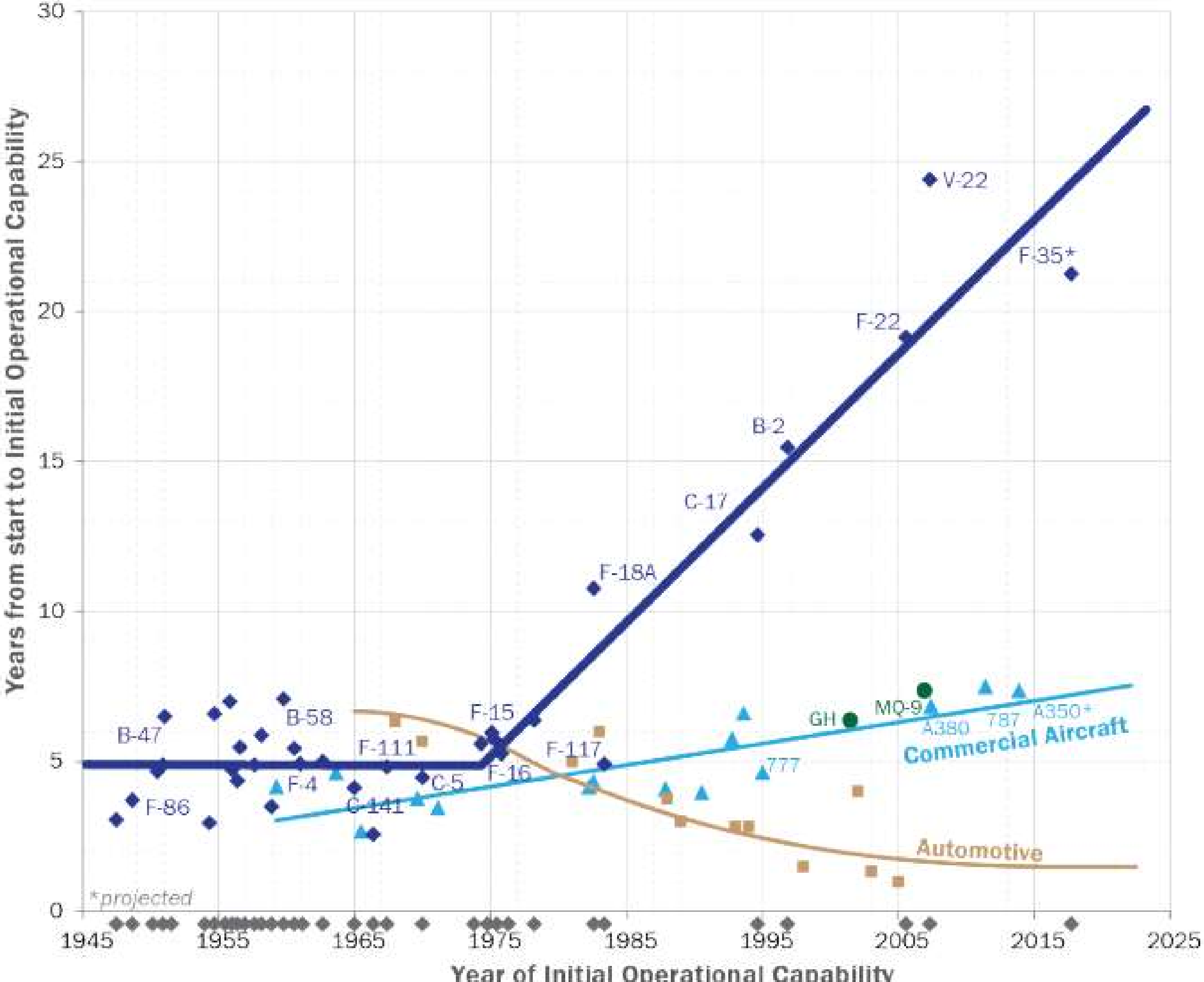

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Cost and time overruns plague new aircraft concepts.

Sources: DARPA 2012 study, RAND MG276, RAND

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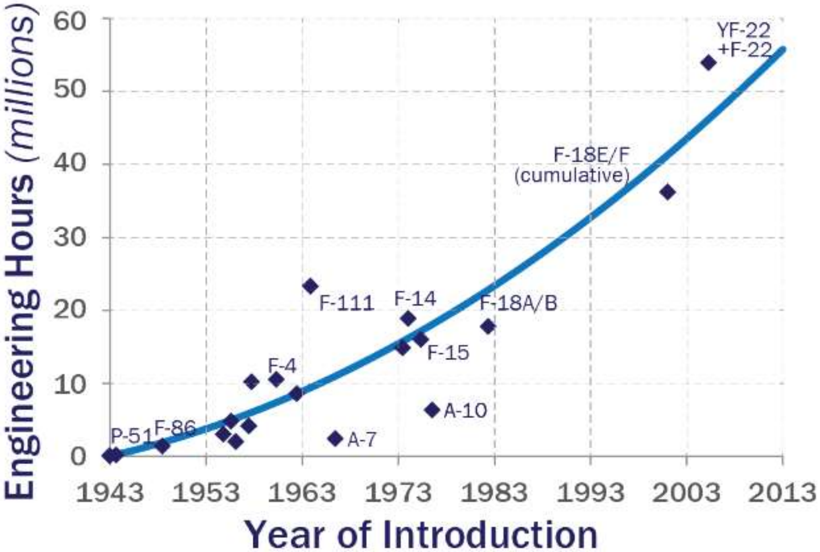
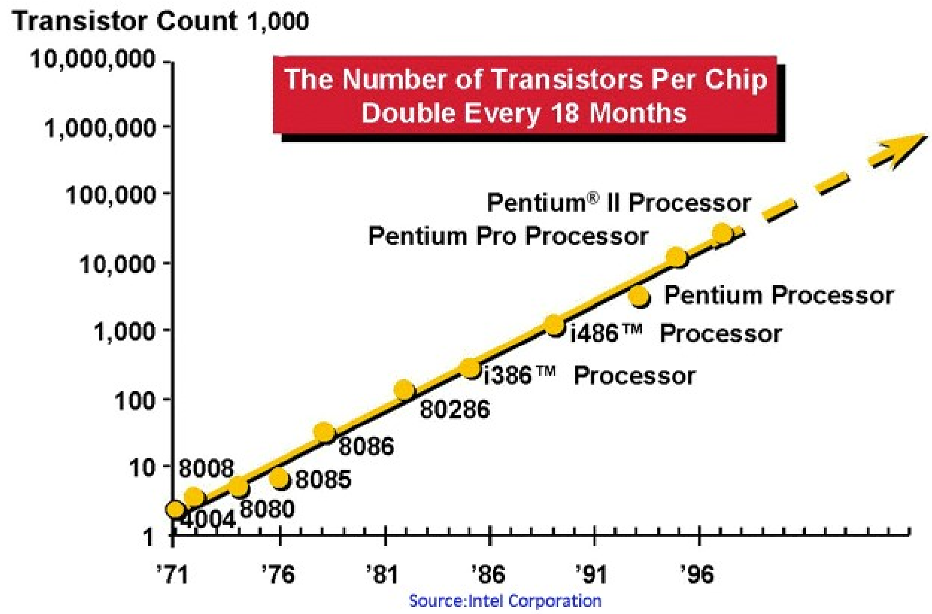
Moore's Law has not made the design process more efficient.



Sources: DARPA 2012 study, RAND MG276, RAND



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We are motivated to address challenges in conceptual design.

Aerospace design problem characteristics:

- Multi-disciplinary
- Tightly coupled
- Data deprived
- Risky

Our chosen approach is to leverage convexity, through geometric and signomial programming using GPkit.

Key takeaway: Signomial programs (SPs) are a competitive method to solve NLPs in engineering design, but better algorithms/heuristics are required.

What to expect

- Broad mathematical overview of log-convexity.
- Advantages of signomial versus geometric programs.
- Heuristics and algorithms to solve SPs.
- Applications and results.
- Challenges in solving SPs.

MATHEMATICAL BACKGROUND

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Geometric programming (GP) is accurate and practical to solve (certain) NLPs.

$$
\begin{aligned}
\text{minimize} \quad & p_0(\boldsymbol{x}) \\
\text{subject to} \quad & p_i(\boldsymbol{x}) \leq 1, \quad i = 1, \ldots, n_p, \\
& m_i(\boldsymbol{x}) = 1, \quad i = 1, \ldots, n_m, \\
& \boldsymbol{x} \in \mathbb{R}^n_{++}, [c, c_k] \in \mathbb{R}^n_{+},
\end{aligned}
$$

$$
m(\boldsymbol{x}) = c \prod_{j=1}^{n} x_j^{a_j},
$$

$$
p(\boldsymbol{x}) = \sum_{k=1}^{K} c_k \prod_{j=1}^{n} x_j^{a_{jk}},
$$

- Advantages:
  - Ability to capture real-world complexity
  - Solution speed
  - Global optimality
  - Sensitivities
- Disadvantages:
  - Stringent formulation
  - Explicit constraints

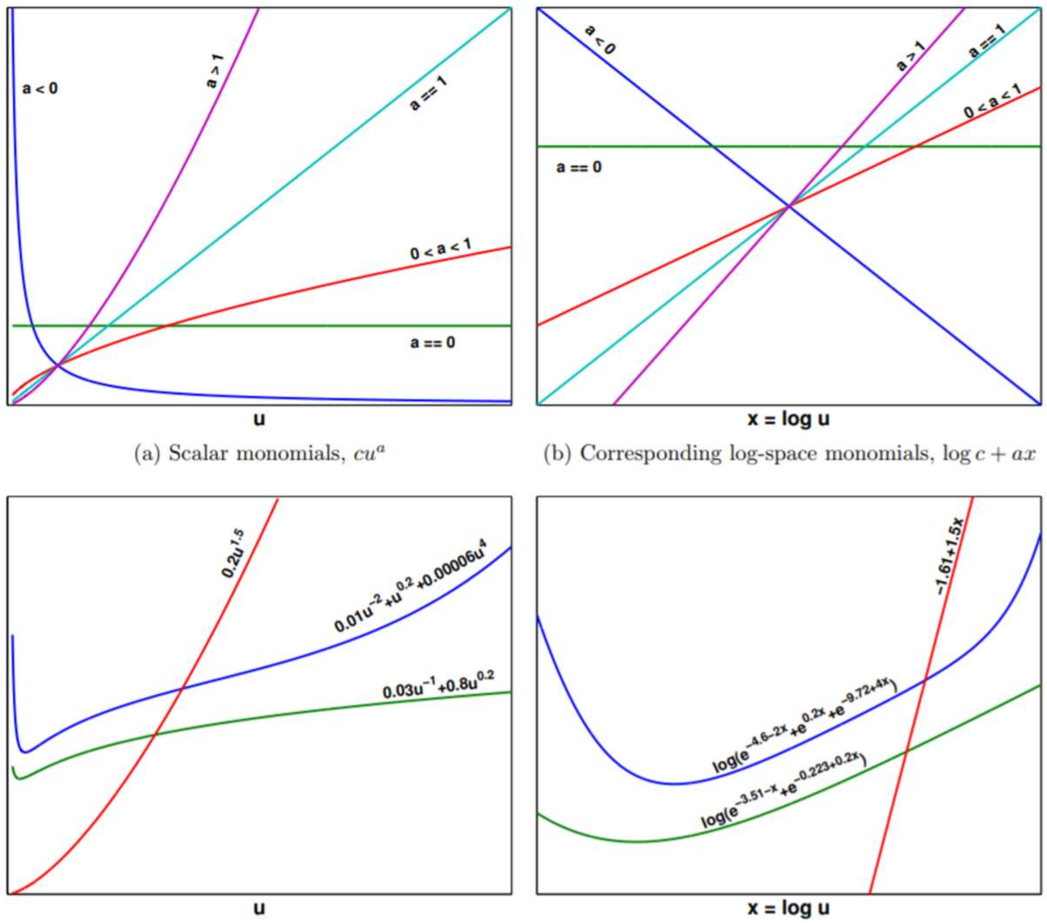
Log-log transformation to turn NLP into convex problem

Geometric form monomial

Exponential form monomial

Geometric form posynomial

Exponential form posynomial



(a) Scalar monomials, cu^a

(b) Corresponding log-space monomials, $\log c + ax$

(c) Scalar posynomials, $\sum_k c_k u^{a_k}$

(d) Corresponding log-space posynomials, $\log \sum_k e^{\log c_k + a_k x}$

*Hoburg, 2013. *Aircraft Design Optimization as a Geometric Program*

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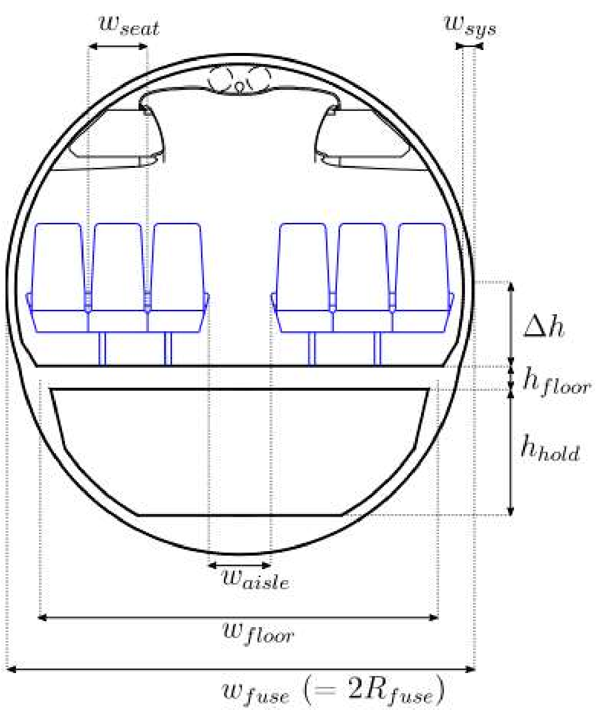
Many engineering constraints are GP compatible.



Describing fuselage configuration:

$$w_{fuse} \geq (SPR)w_{seat} + w_{aisle} + 2w_{sys}$$

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Kirschen, P. G., York, M. A., Ozturk, B., and Hoburg, W. W., “Application of Signomial Programming to Aircraft Design,” *Journal of Aircraft*, 2017, pp. 1–23.

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Data can be fit with posynomials.

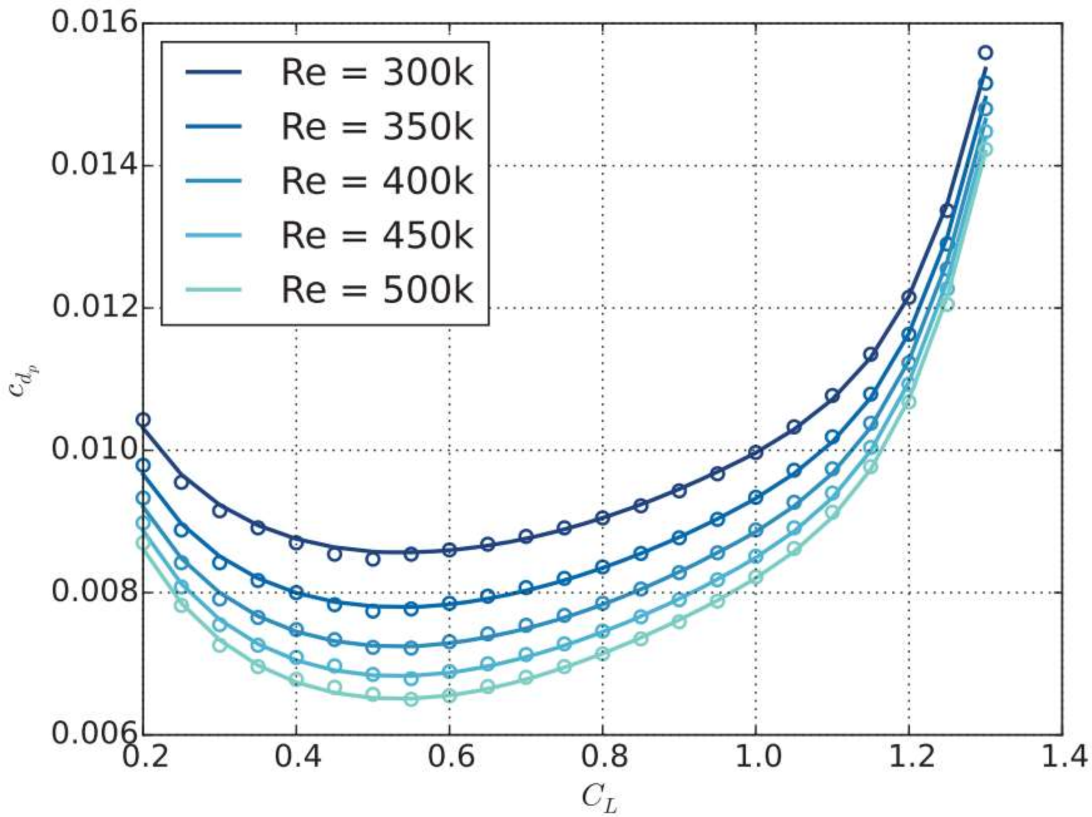

Fig. 7 Posynomial fit (solid lines) to XFOIL data (circles). Log-space rms error = 0.00489.

Burton, M., and Hoburg, W., "Solar and Gas Powered Long-Endurance Unmanned Aircraft Sizing via Geometric Programming," *Journal of Aircraft*, vol. 55, 2017, pp. 212–225.

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GPs have been used to design the Jungle Hawk Owl (JHO).

Source: MIT News

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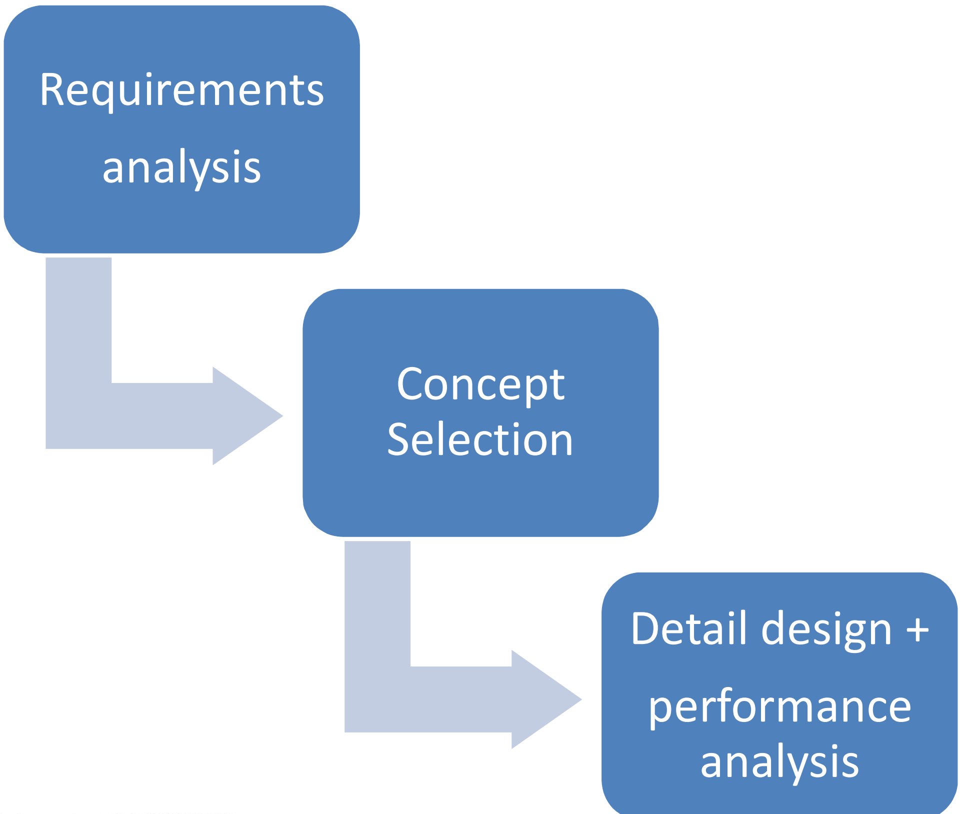
GP was used in every step of the design process.



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Burton, M. J., and Hoburg, W. W., "Solar-Electric and Gas Powered, Long-Endurance UAV Sizing via Geometric Programming," *18th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference*, 2017, pp. 1–26.

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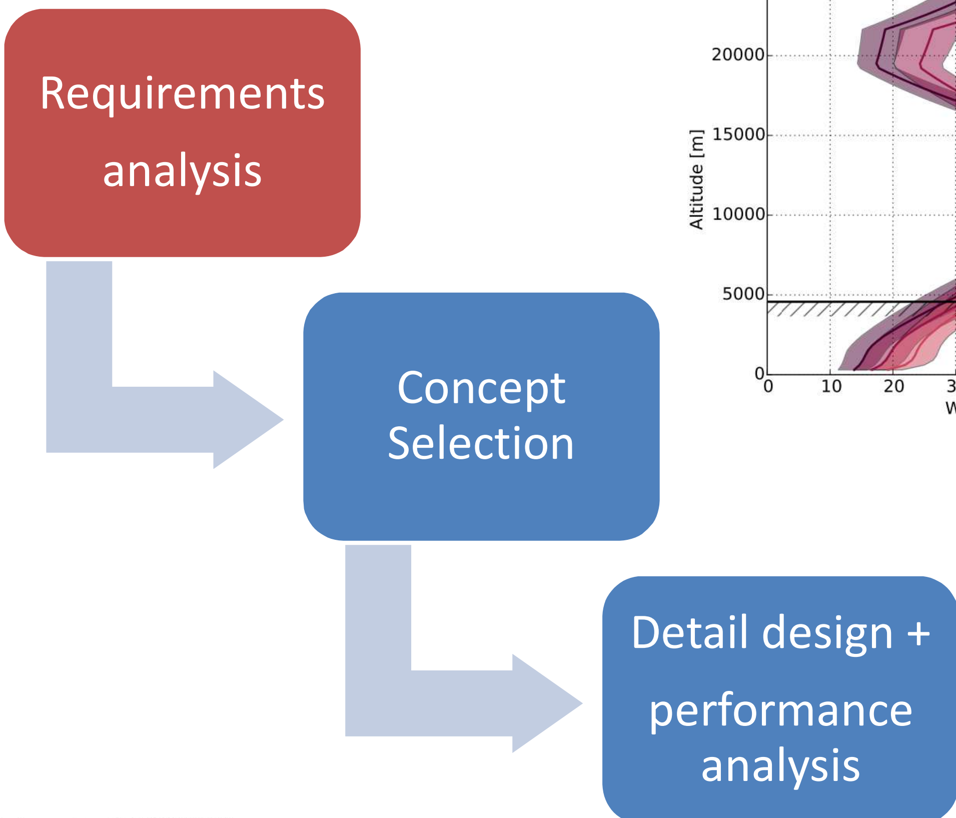
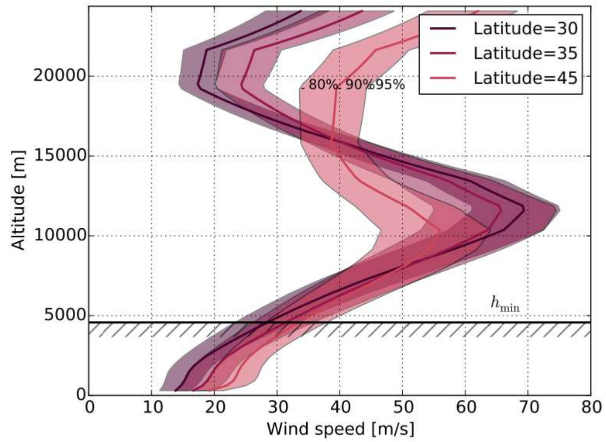
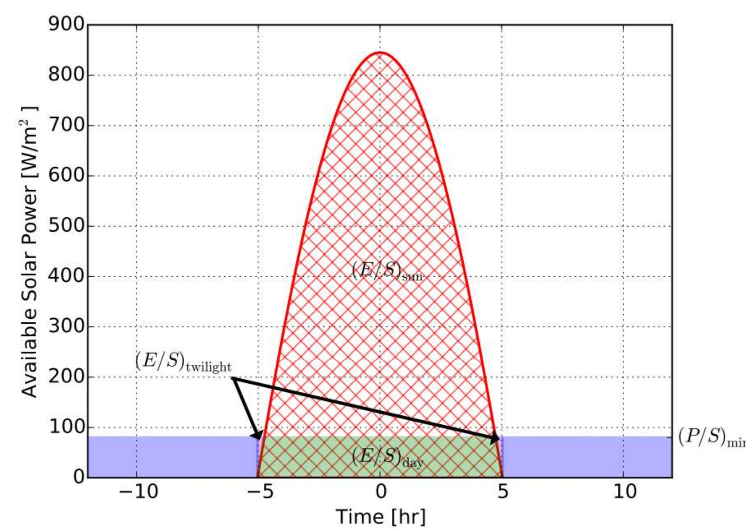
GP was used to understand aircraft ‘limiters’.







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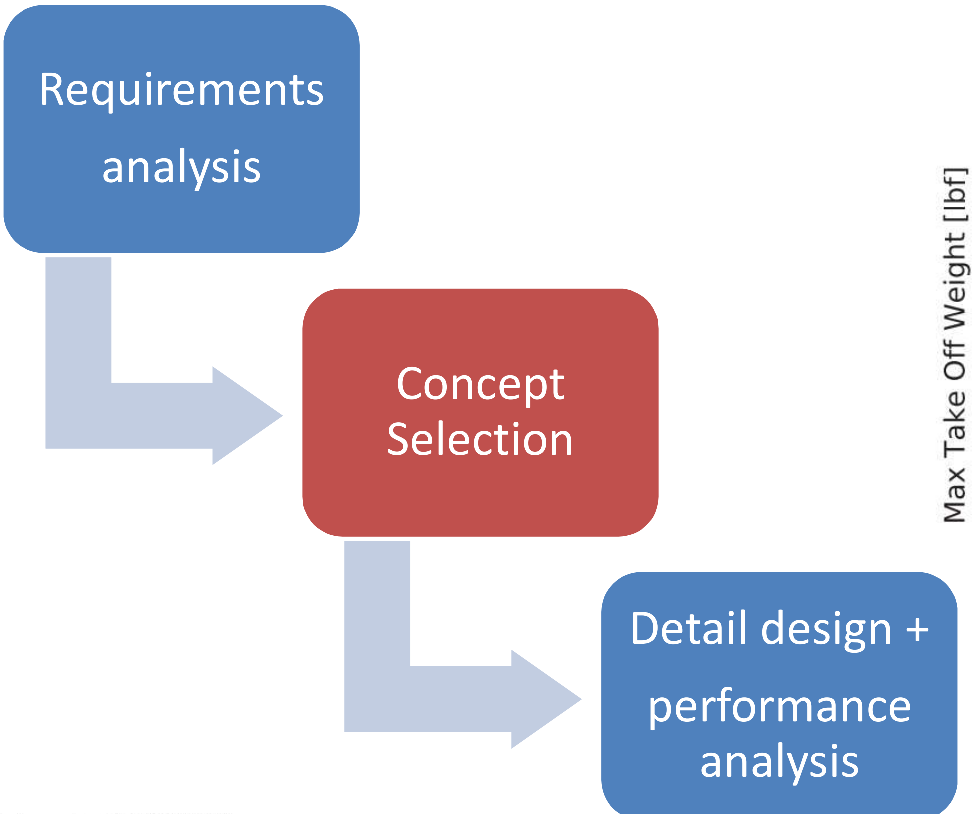
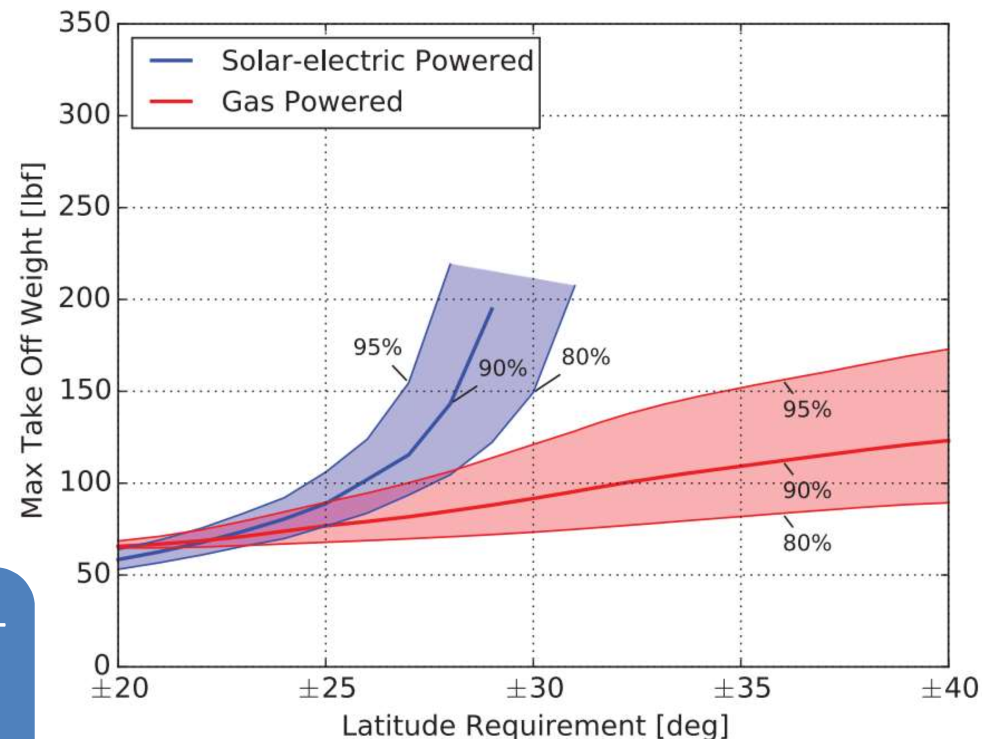
Burton, M. J., and Hoburg, W. W., “Solar-Electric and Gas Powered, Long-Endurance UAV Sizing via Geometric Programming,” *18th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference*, 2017, pp. 1–26.

Gas-powered aircraft concept proves superior to solar.





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Burton, M. J., and Hoburg, W. W., "Solar-Electric and Gas Powered, Long-Endurance UAV Sizing via Geometric Programming," *18th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference*, 2017, pp. 1–26.

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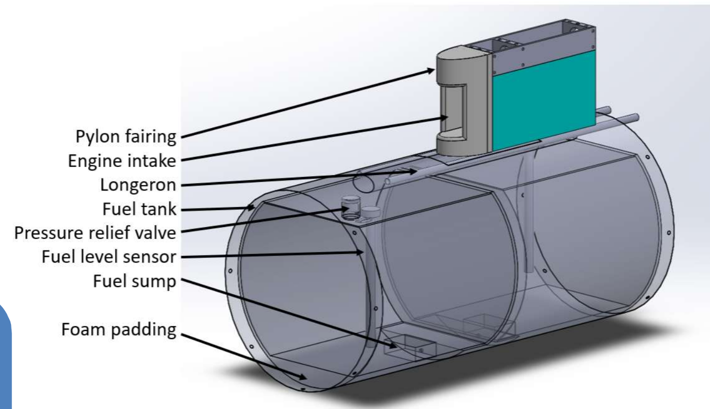
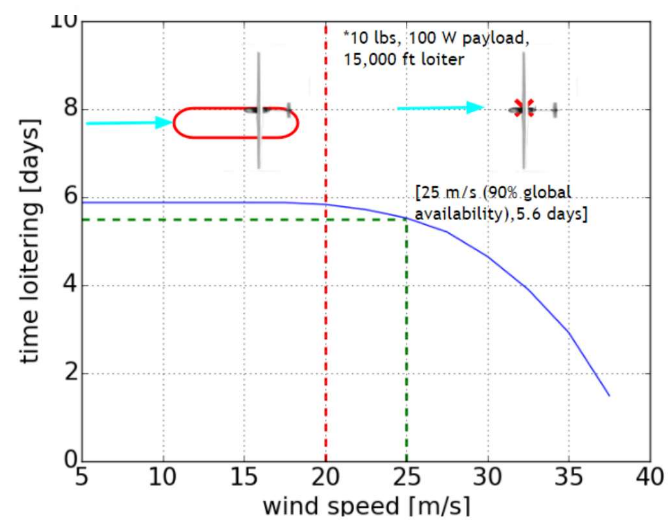
GPs is used to evaluate performance as detailed design decisions are made.

Requirements analysis

Concept Selection

Detail design + performance analysis





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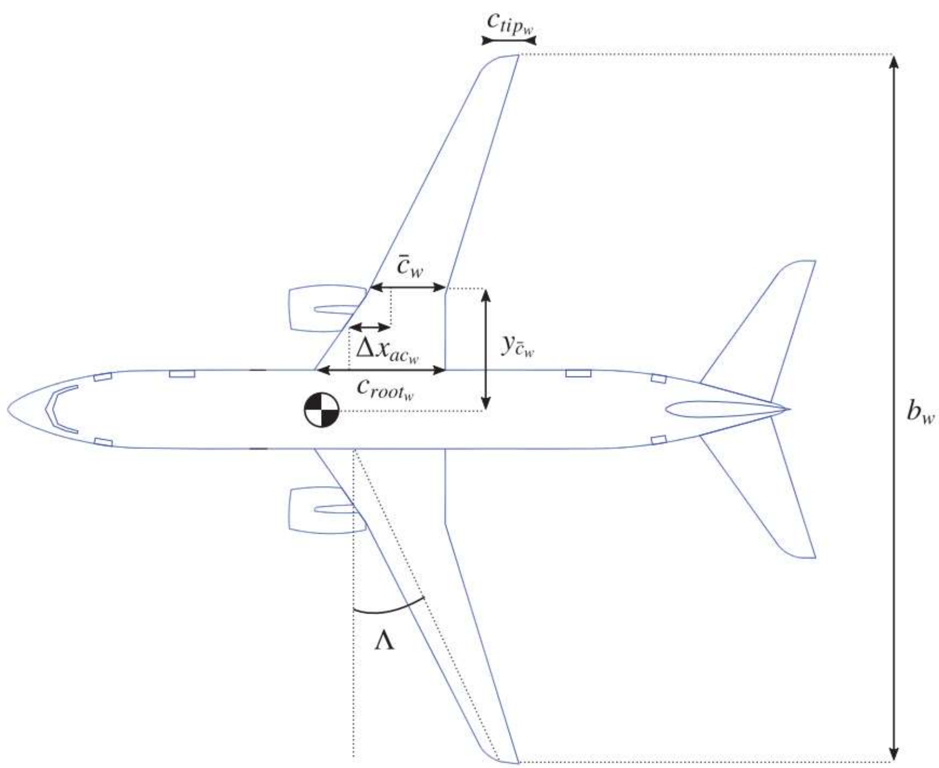
Burton, M. J., and Hoburg, W. W., “Solar-Electric and Gas Powered, Long-Endurance UAV Sizing via Geometric Programming,” *18th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference*, 2017, pp. 1–26.

Some constraints are not GP-compatible.



Constraining wing root bending moment:

$$M_r c_{\mathrm{root}_w} \geq \left(L_{w_{\max}} - N_{\mathrm{lift}}\left(W_{\mathrm{wing}} + f_{\mathrm{fuel,wing}} W_{\mathrm{fuel}_{\mathrm{total}}}\right)\right) \cdot \left(\frac{b_w^2}{12 S_w}\left(c_{\mathrm{root}_w} + 2c_{\mathrm{tip}_w}\right)\right) - N_{\mathrm{lift}} W_{\mathrm{eng}} y_{\mathrm{eng}}$$

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Signomial Programs are more general, and expand the scope of physics we can handle…

Geometric program (GP):

- Log-convex
- Globally optimal
- No initial guesses
- Solved by IP, SQP etc.

$$\begin{aligned} \text{minimize } & f_0(\mathbf{x}) \\ \text{subject to } & f_i(\mathbf{x}) \leq 1, i = 1, ..., m \\ & g_i(\mathbf{x}) = 1, i = 1, ..., p \end{aligned}$$

Signomial program (SP):

- Non-log-convex (difference of convex)
- Locally optimal
- Requires an initial guess
- Solved as a sequence of GPs

$$\begin{aligned} \text{minimize } & f_0(\mathbf{x}) \\ \text{subject to } & f_i(\mathbf{x}) - h_i(\mathbf{x}) \leq 0, i = 1, ..., m \end{aligned}$$

…albeit with loss of mathematical guarantees.

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Formulated in: GPkit

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A number of papers expand on SP-compatible modeling…

Conceptual Engineering Design and Optimization Methodologies using Geometric Programming

by

Berk Öztürk

Submitted to the Department of Aeronautics and Astronautics on February 1st, 2018, in partial fulfillment of the requirements for the degree of Master of Science in Aeronautics and Astronautics

Solar and Gas Powered Long-Endurance Unmanned Aircraft Sizing via Geometric Programming

Michael Burton* and Warren Hoburg†
Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

DOI: 10.2514/1.C034405

Efficient Aircraft Multidisciplinary Design Optimization and Sensitivity Analysis via Signomial Programming

Martin A. York,* Berk Öztürk,† Edward Burnell,‡ and Warren W. Hoburg§
Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

DOI: 10.2514/1.J057020

Turbofan Engine Sizing and Tradeoff Analysis via Signomial Programming

Martin A. York,* Warren W. Hoburg,† and Mark Drela‡
Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

DOI: 10.2514/1.C034463

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ALGORITHMS

For GPs, the power of log transformation is clear.

		Without log transformation						With log transformation		
		No analytical gradients			Analytical gradients			No analytical gradients		
Solver type	Initial guess	$f(x)$ [N]	t [s]	n [-]	$f(x)$ [N]	t [s]	n [-]	$f(x)$ [N]	t [s]	n [-]
IP	All 1's	303.14	9.8	2725	1.2802e-06(e)	1436.8	300000	303.07	0.2	28
IP	Near opt.	303.14	0.2	105	303.14	0.2	90	303.07	0.2	14
IP	OM, floor	0.0001601(i)	852.2	227857	0.00016007(e)	1225.3	300000	303.07	0.2	19
IP	OM, round	303.14	53.2	11562	593.76	37.7	10530	303.07	0.1	20
IP	OM, mix	9.9955e-07	70.0	24621	303.14	17.4	5039	303.07	0.1	19
SQP	All 1's	303.14	0.1	94	303.14	0.1	274	303.07	0.1	20
SQP	Near opt.	304.95	0.0	23	304.95	0.0	23	303.07	0.1	9
SQP	OM, floor	337.79	0.2	83	337.79	0.0	83	303.07	0.1	12
SQP	OM, round	438.66	1.2	653	438.66	0.1	83	303.07	0.1	11
SQP	OM, mix	337.85	0.1	72	337.85	0.0	72	303.07	0.1	12

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Borrowed from: Kirschen, P. G., and Hoburg, W. W., "The Power of Log Transformation: A Comparison of Geometric and Signomial Programming with General Nonlinear Programming Techniques for Aircraft Design Optimization," AIAA SciTech 2018.

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…but SPs are non-log-convex. Simple to (locally) convexify.

Signomials are difference of posynomials.

$$s(\mathbf{x}) \leq 0$$
$$p(\mathbf{x}) - q(x) \leq 0$$
$$p(\mathbf{x}) \leq q(\mathbf{x})$$

Monomial approx. of RHS makes signomial into posynomial.

$$p(\mathbf{x}) \leq \hat{q}(\mathbf{x};\mathbf{x}^0)$$
$$\frac{p(\mathbf{x})}{\hat{q}(\mathbf{x};\mathbf{x}^0)} \leq 1$$

The best local monomial approx. of posynomial is known.

$$\hat{q}(\mathbf{x})|_{\mathbf{x}^0} = q(\mathbf{x}^0)\prod_{i=1}^{n}\left(\frac{x_i}{x_i^0}\right)^{a_i}$$
$$a_i = \frac{x_i^0}{q(\mathbf{x}^0)}\frac{\partial q}{\partial x_i}$$

Borrowed from: Kirschen, P. G., and Hoburg, W. W., "The Power of Log Transformation: A Comparison of Geometric and Signomial Programming with General Nonlinear Programming Techniques for Aircraft Design Optimization," AIAA SciTech 2018.
Theory: Lipp, T., and Boyd, S., "Variations and extension of the convex – concave procedure," *Optimization and Engineering*, vol. 17, 2016, pp. 263–287.

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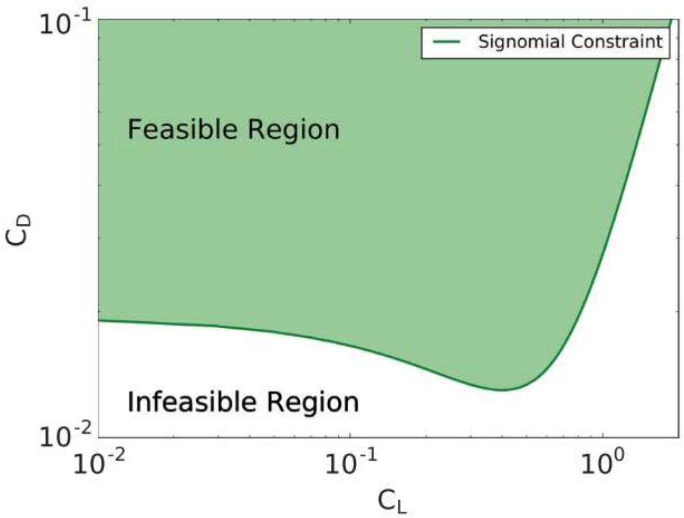
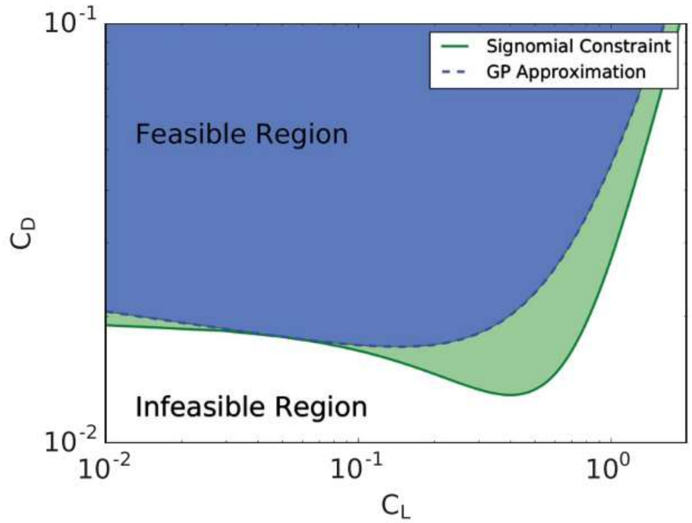
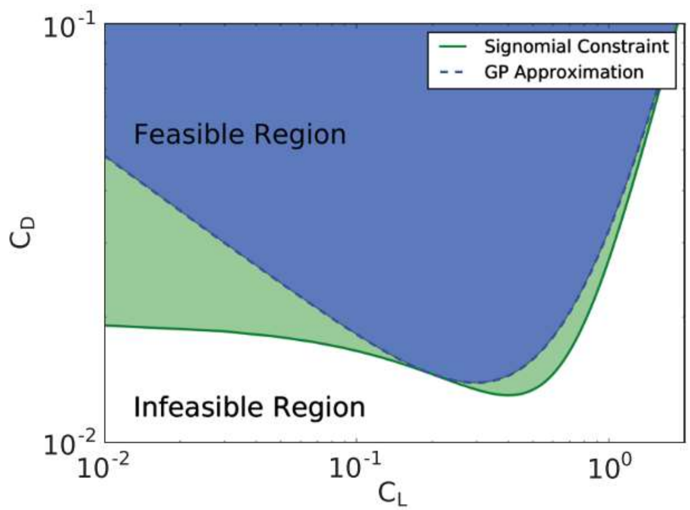


a) Non-convex signomial inequality drag constraint



b) Convex approximation about $C_L = 0.05$

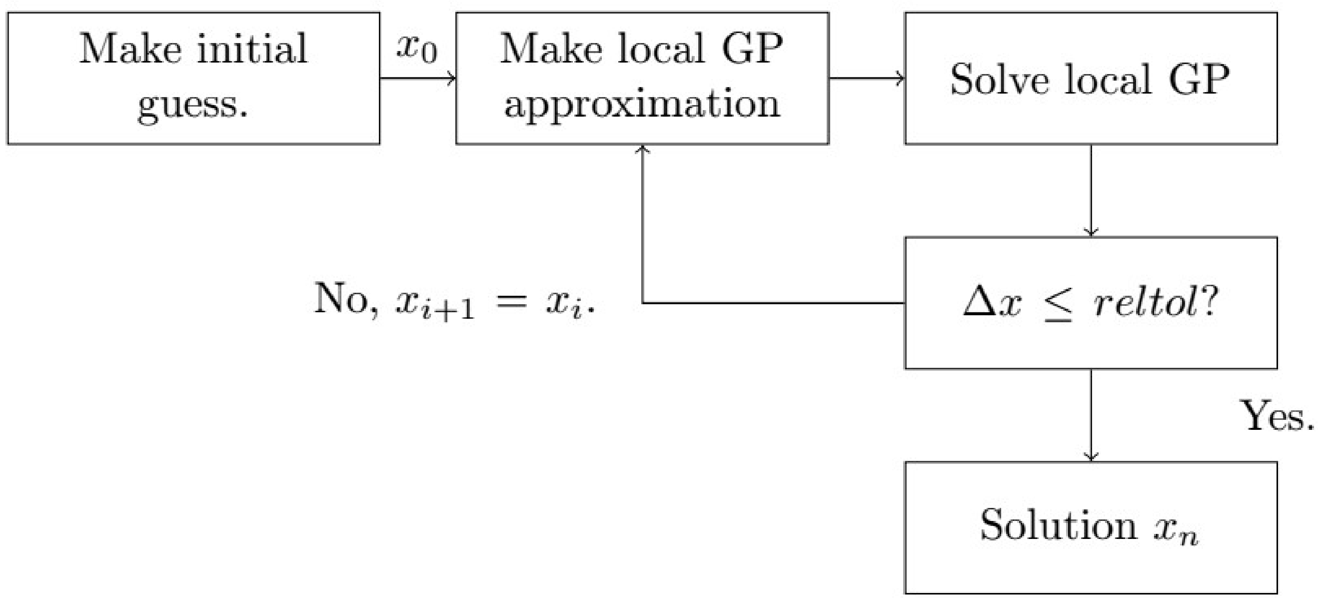


c) Convex approximation about $C_L = 0.20$

York, M. A., Öztürk, B., Burnell, E., and Hoburg, W. W., "Efficient Aircraft Multidisciplinary Design Optimization and Sensitivity Analysis via Signomial Programming," pp. 1–16.

Posynomial approx. of signomial is *in the interior* of feasible region of the signomial.

SPs can be solved as a sequence of GPs.



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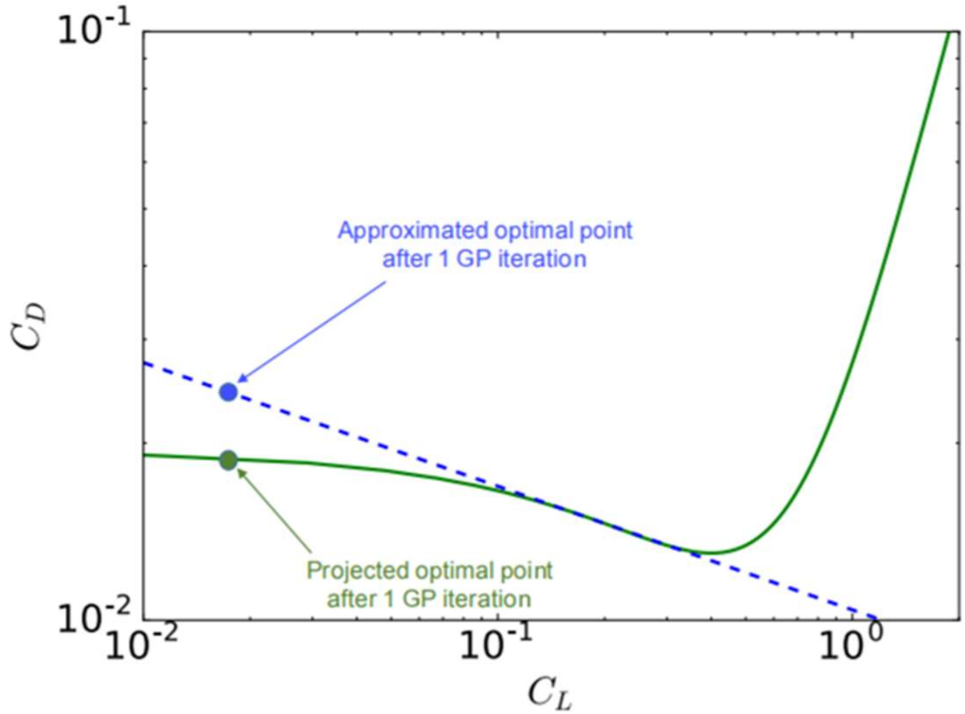
Relaxations help if initial guess/lack of sparsity are problematic.

$$\begin{aligned} \text{minimize} \quad & f(\mathbf{x}, \mathbf{u}) \\ \text{s.t.} \quad & p_i(\mathbf{x}, \mathbf{u}) - q_i(\mathbf{x}, \mathbf{u}) \le 0, \;\; i = 1, \ldots, m \end{aligned}$$

Relaxed constants

$$\begin{aligned} \text{minimize} \quad & \left[\prod_{i=1}^{m} s_i^{\alpha}\right] f(\mathbf{x}, \mathbf{v}) \\ \text{s.t.} \quad & p_i(\mathbf{x}, \mathbf{v}) - q_i(\mathbf{x}, \mathbf{v}) \le 0, \;\; i = 1, \ldots, m \\ & \frac{u_i}{s_i} \le v_i \le s_i u_i, \;\; s_i \ge 1, \;\; i = 1, \ldots, m \end{aligned}$$

Approximations of non-log-affine equalities are somewhat tractable.



- Only log-affine equalities (monomials in geometric problem) are ever convex.
- Work by Opgenoord shows monomial approx. works with signomial qualities.
- However, there are limits…

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Opgenoord, M. M. J., Cohen, B. S., and Hoburg, W. W., "Comparison of Algorithms for Including Equality Constraints in Signomial Programming," *ACDL Technical Report, TR-2017-1*, 2017, pp. 1–23.

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Equalities are a last resort.

- To be used when the pressure on variables is not clear.

$$\rho = \frac{P}{RT}, \qquad \left(\frac{P}{P_{SL}}\right)^{LR/g} = \frac{T}{T_{SL}}, \qquad T_{SL} = T + Lh$$

Monomial equalities,
Implemented without approximation

No, signomial equality.

Possibly a signomial inequality relaxation?

Example from : York, M. A., Hoburg, W. W., and Drela, M., "Turbofan Engine Sizing and Tradeoff Analysis via Signomial Programming," *Journal of Aircraft*, vol. 55, 2018.

The log transformation is also essential for SPs.

		Without log transformation						With log transformation		
		No analytical gradients			Analytical gradients			No analytical gradients		
Solver type	Initial guess	$f(x)$ [N]	t [s]	n [-]	$f(x)$ [N]	t [s]	n [-]	$f(x)$ [N]	t [s]	n [-]
IP	All 1's	0.00029284(i)	316.6	73654	9.5991e-05(e)	1232.5	300000	4536.2	0.4	103
IP	Near opt.	4543.6	10.0	1939	4543.6	5.8	1694	4536.2	0.3	61
IP	OM, floor	4543.6	9.7	2006	4543.6	28.4	5025	4536.2	0.4	97
IP	OM, round	4543.6	300.5	55821	11062	429.5	115141	4536.2	0.3	68
SQP	All 1's	21.3(i)	0.1	13	-2.5512e-05(i)	0.1	32	4536.2	0.0	51
SQP	Near opt.	4547.9	0.1	34	4547.9	0.1	48	4536.2	0.0	25
SQP	OM, floor	3.4751e+06	3.5	772	1.0339e+06	1.0	486	4536.2	0.1	43
SQP	OM, round	5132.1	0.4	136	5619.9(i)	0.0	2	4536.2	0.1	30

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Borrowed from: Kirschen, P. G., and Hoburg, W. W., "The Power of Log Transformation: A Comparison of Geometric and Signomial Programming with General Nonlinear Programming Techniques for Aircraft Design Optimization," AIAA SciTech 2018.

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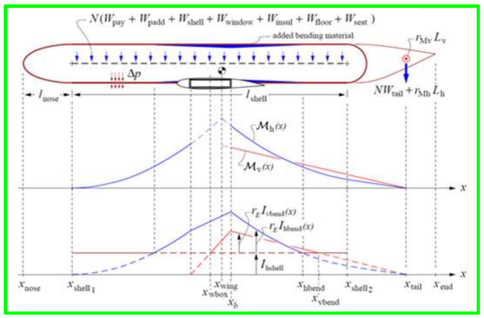
APPLICATIONS

SP models can be arbitrarily complex.



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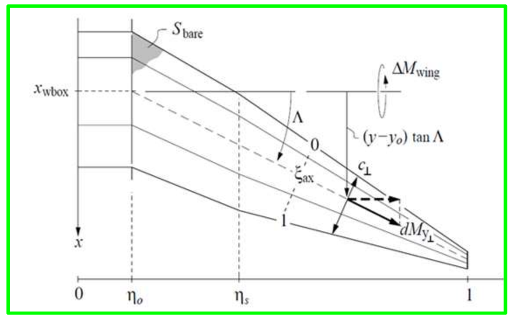

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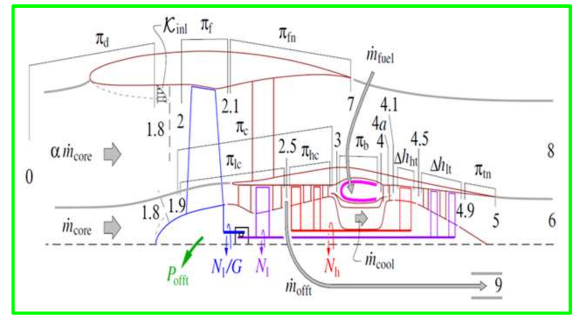

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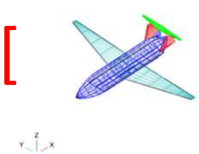

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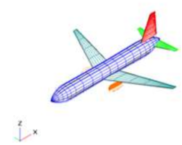


| Mission + sizing constraints]

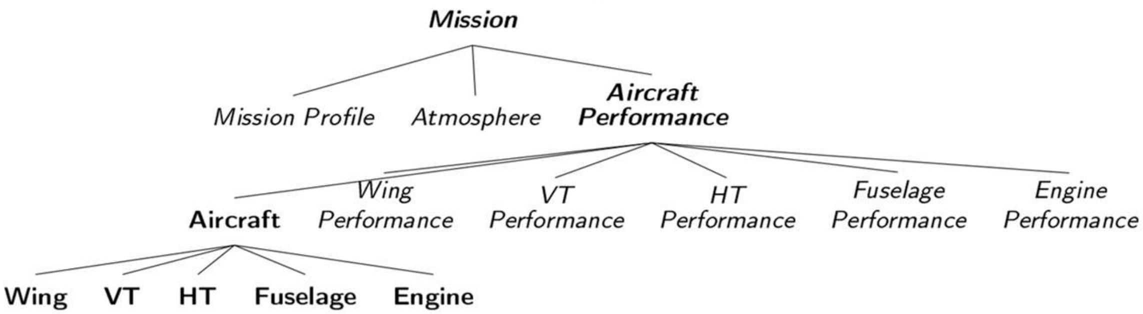
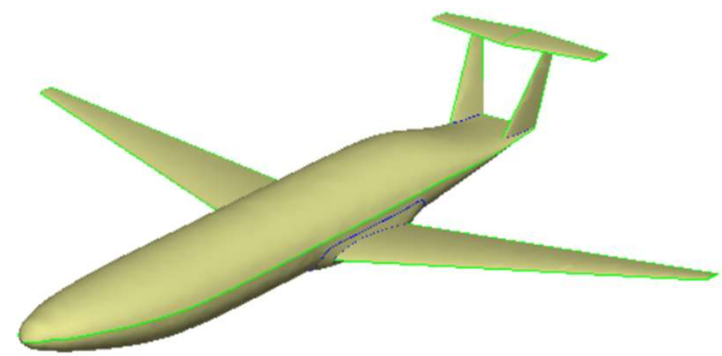


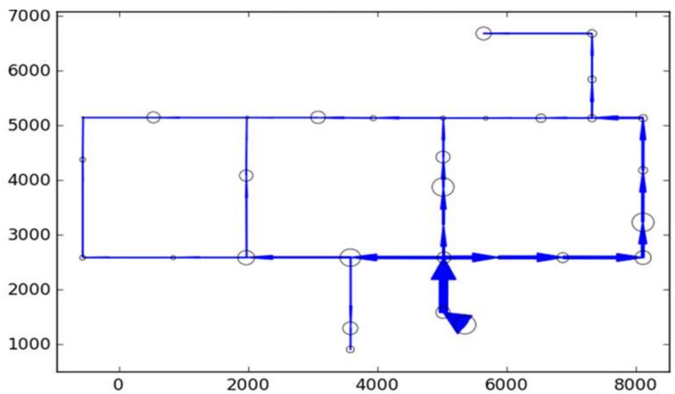
Figure 4-1: Hierarchy of the presented aircraft model. Models that include sizing variables are bolded while models that include performance variables are italicized. There are models that contain both kinds of variables.



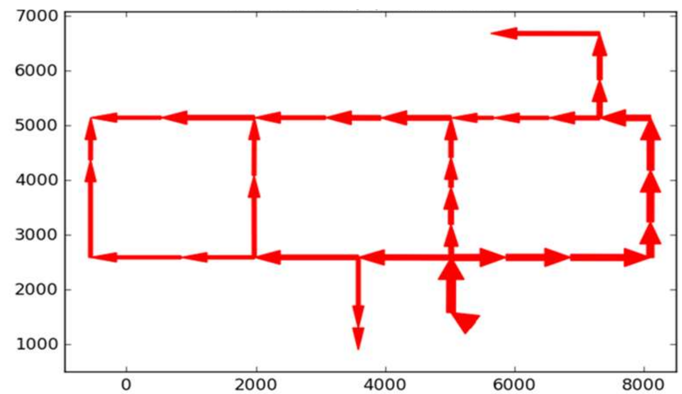
York, M. A., Öztürk, B., Burnell, E., and Hoburg, W. W., "Efficient Aircraft Multidisciplinary Design Optimization and Sensitivity Analysis via Signomial Programming," pp. 1–16.

GPs → trees. SPs → graphs.

Normalized flow rate



Normalized pipe diameter



Hanoi water distribution benchmark from University of Exeter Centre for Water Systems

- Recent work to expand scope of [Perelman, 2015].
- Conservation of mass and momentum. Non-linear edge costs.
- Graphs can be scaled arbitrarily to test future algorithms.

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Perelman, L. S., and Amin, S., "Control of tree water networks: A geometric programming approach," *American Geophysical Union*, 2015.

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Optimization under uncertainty

- [Saab, 2018] used principles from robust LP to formulate approximate robust GPs.
- We expanded framework to SPs, with promising results.

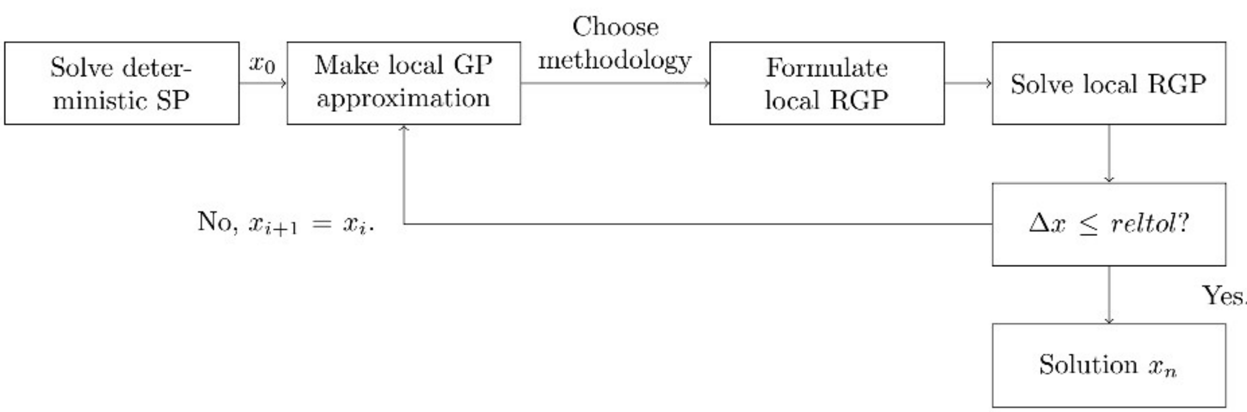
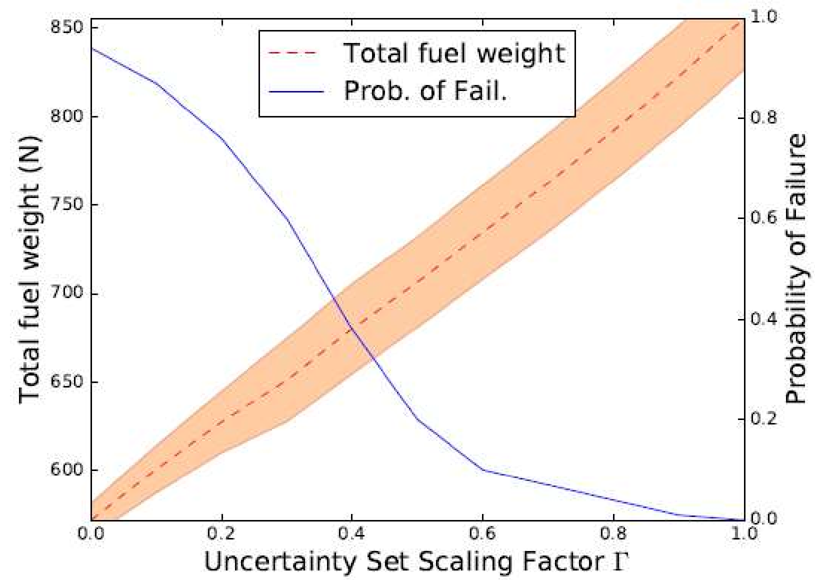


Figure 3: A block diagram showing the steps of solving an RSP.



(b) Elliptical Uncertainty Set

[1] Saab, A., Burnell, E., and Hoburg, W. W., "Robust Designs via Geometric Programming," *Society for Industrial and Applied Mathematics*, 2018, pp. 1–23.
[2]Ozturk, B., and Saab, A., "Optimal Aircraft Design Decisions under Uncertainty via Robust Signomial Programming," *AIAA Aviation 2019 Forum*, 2019.

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Overview: SPs are powerful in conceptual engineering design, but focused research is required on:

- Optimality guarantees, or good lower bounds
- Approximations for equalities
  - Quantify when/why they are problematic.
  - Perhaps give U/L approximations another shot.
- Better solution heuristics given naïve initial guesses.

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Please find our engineering design optimization packages and models at:
https://github.com/convexengineering

This work is powered by:

GPkit: .../gpkit
gpfit: .../gpfit
robust: .../robust (in development)

Mosek Version 8.1.0.80

Looking forward to your questions!

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BACK-UP SLIDES

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We are motivated to address challenges in conceptual design.

Problem characteristic	Common design practice	Design outcome
Multi-disciplinary	Collaborative optimization	Suboptimal
Data deprived	Heavy reliance on experienced engineers	Dogmatic
Risky	Non-rigorous uncertainty capturing	Conservative

Office [3]1

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Slide 36

Office [3]1 I agree with the claims you make here, and I like where you're going, but I think this slide ends up being weak because it comes across as being your claims/opinions not backed up by any data or sources. I wouldn't create such a linear mapping from "problem characteristic" to "design outcome" and group into three categories. Rather, I'd turn this into a broad landscape of optimization methods (UQ, MDO, etc etc) and focus more on the challenges (multidisciplinary, uncertainty, *non-convexity*, etc) than on the algorithms. Your previous two slides already convince me that there's a problem/challenge. Story wise, this slide can focus on helping your optimization-inclined audience to understand what mathematical challenges are driving the problem.

Microsoft Office User, 8/2/2019

Key question:
how to improve the conceptual design process?

- Make the constraints as general as possible.
- Solve data-deprived problems through physics:
  - Aim to understand tradeoffs, not just figures.
  - Leverage data where possible.
- Capture uncertainty in a tractable manner.

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Other constraints can be approximated.

Geometric averages

$$W_{\text{ave}} = \sqrt{W_{\text{initial}} W_{\text{final}}}$$

Taylor expansions

$$\frac{W_{\text{fuel}_i}}{W_{i+1}} \geq z_{\text{bre}_i} + \frac{z_{\text{bre}_i}^2}{2} + \frac{z_{\text{bre}_i}^3}{6} + \frac{z_{\text{bre}_i}^3}{24}$$

Dummy variables

$$1 \geq \frac{(z_{\text{CG}} + l_m)^2 (y_m^2 + B^2)}{(\Delta x_n y_m \tan(\psi))^2}$$

And so on…

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Equality relaxations make problem GP-compatible

Traditional: potential and flow functions

$$\underset{\mathbf{H},\beta,\gamma}{\text{minimize}} \quad \sum_{i \in E_{pu}} c_{1k} q_k \beta_k^{m_{1k}},$$

New approach: relaxed potential and flow functions

$$\underset{\mathbf{H},\beta,\gamma}{\text{minimize}} \quad \sum_{k \in E_{pu}} c_{1k} q_k \beta_k^{m_{1k}} + \sum_{k \in E_v} c_{2k} \gamma_k^{-m_{2k}} + \sum_{i \in N_d} c_{3i} H_i^{-m_{3i}},$$

Traditional		New approach
$H_j = \gamma_k H_i \quad \forall k \in E_v,$	Valve operation	$H_j = \gamma_k H_i \quad \forall k \in E_v,$
$H_j + h_k = H_i \quad \forall k \in E_p,$	Flow pressure loss	$H_j + h_k \leq H_i \quad \forall k \in E_p,$
$H_j = \beta_k H_i \quad \forall k \in E_{pu},$	Pump operation	$H_j = \beta_k H_i \quad \forall k \in E_{pu},$
$\underline{H}_i \leq H_i \leq \bar{H}_i \quad \forall i \in N_d,$	Head pressure bounds	$\underline{H}_i \leq H_i \leq \bar{H}_i \quad \forall i \in N_d,$
$1 \leq \beta_k \leq \bar{\beta}_k \quad \forall k \in E_{pu},$	Pump setting bounds	$1 \leq \beta_k \leq \bar{\beta}_k \quad \forall k \in E_{pu},$
$0 \leq \gamma_k \leq 1 \quad \forall k \in E_v,$	Valve setting bounds	$0 \leq \gamma_k \leq 1 \quad \forall k \in E_v,$
$h_k = R_k q_k^{\alpha} \quad \forall k \in E_p$	Flow pressure loss	$h_k = R_k q_k^{\alpha} \quad \forall k \in E_p$

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SIGNOMIAL PROGRAMMING UNDER UNCERTAINTY

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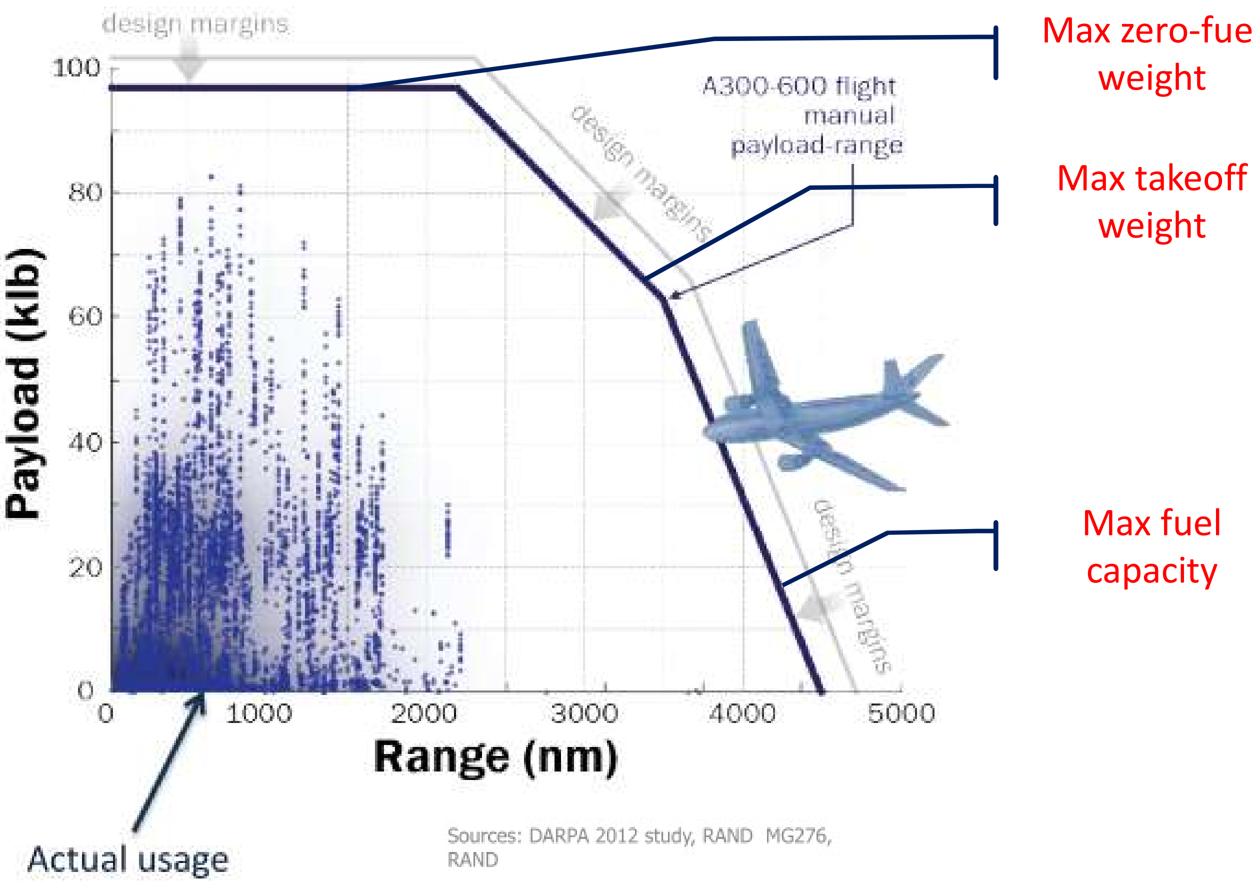
Inability to handle parametric uncertainty results in conservative aerospace designs.



How can we use principles of **robust optimization** to improve on legacy design methods?

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Idea: tractable optimization under uncertainty using SPs.

- Combine principles from robust linear programming with GPs.
  - Separate posynomials into two-term posynomials, WLOG.
  - Robustify conservative PWL approximation of posynomials.
- Augment SP heuristic with robust approximations of GPs.

The robust counterpart transforms OUU to deterministic optimization problem.

Optimization over:

$$\begin{aligned} &\min\ f_0(x) \\ &\text{s.t. } f_i(x,u) \le 0,\ \forall u \in \mathcal{U},\ i = 1,\dots,n \end{aligned}$$

Infinite number of constraints

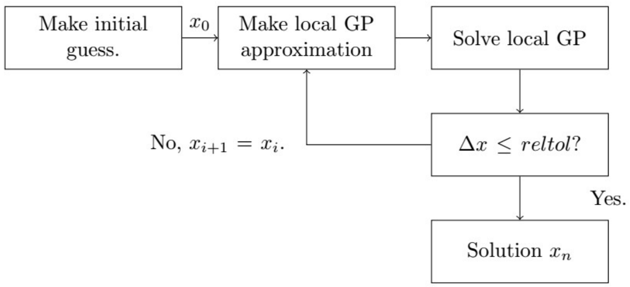
$$\begin{aligned} &\min\ f_0(x) \\ &\text{s.t. } \max_{u\in\mathcal{U}} f_i(x,u) \le 0,\ i = 1,\dots,n \end{aligned}$$

Finite number of constraints

$$\begin{aligned} &\min\ f_0(x) \\ &\text{s.t. } \max_{u} f_i(x,u) \le 0,\ i = 1,\dots,n \\ &\qquad \|u\| \le \Gamma \end{aligned}$$

A well-defined set

We augment the SP solution heuristic.



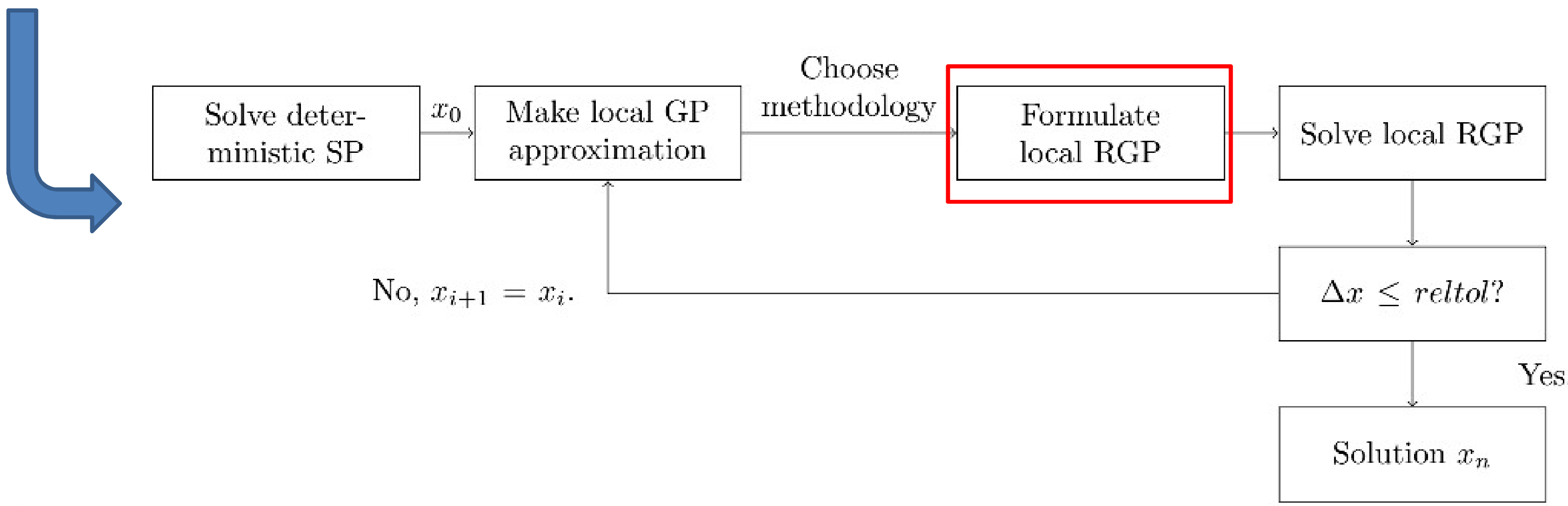


Figure 3: A block diagram showing the steps of solving an RSP.

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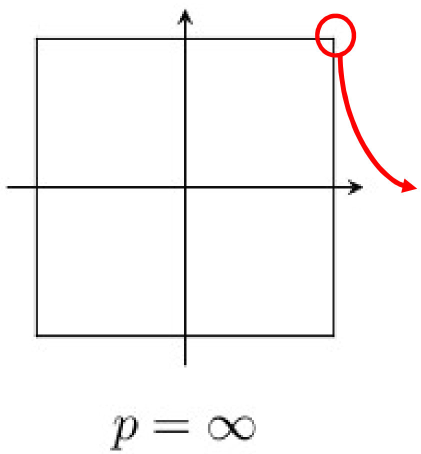
Uncertainty sets considered

Box (L-∞ norm)

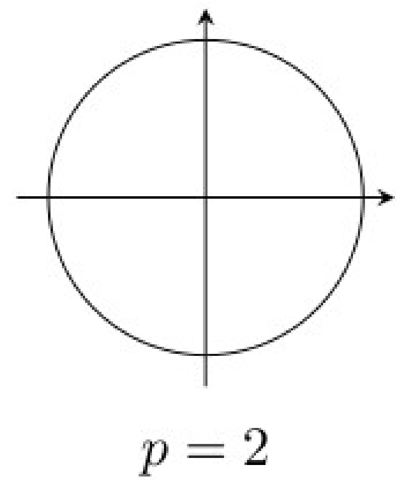
More conservative than margins.



Margins optimize on a corner of the hypercube!

Elliptical (L-2 norm)

A less conservative candidate!



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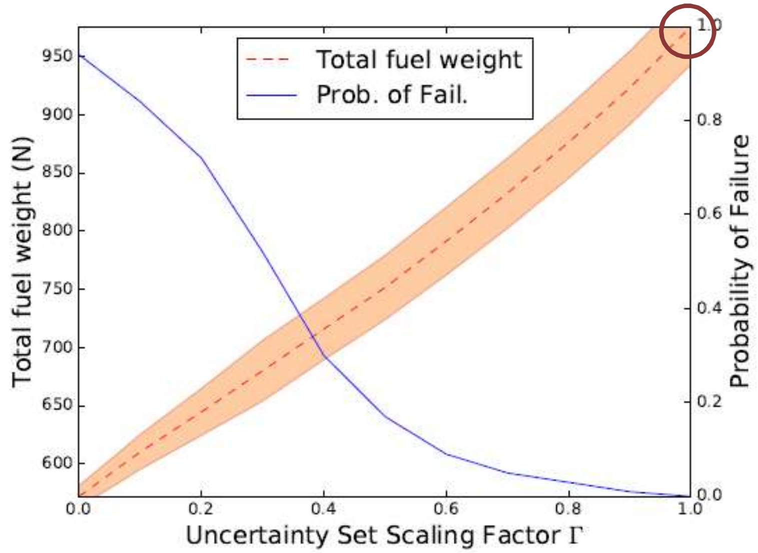
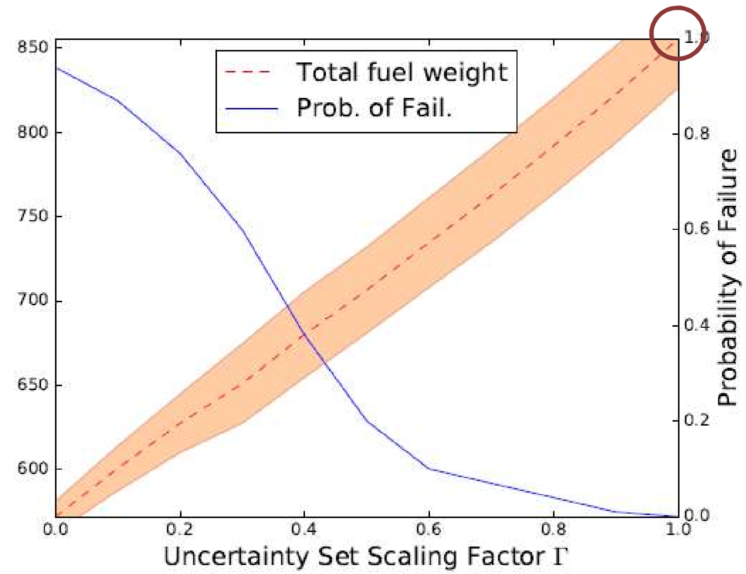
RSP successfully mitigates probability of failure.



(a) Box Uncertainty Set



(b) Elliptical Uncertainty Set

For Γ = 1, the elliptical design spends 14% less fuel than the box design, while protecting against the same uncertainty!

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