Optimal Aircraft Design Decisions Under Uncertainty via Robust Signomial Programming

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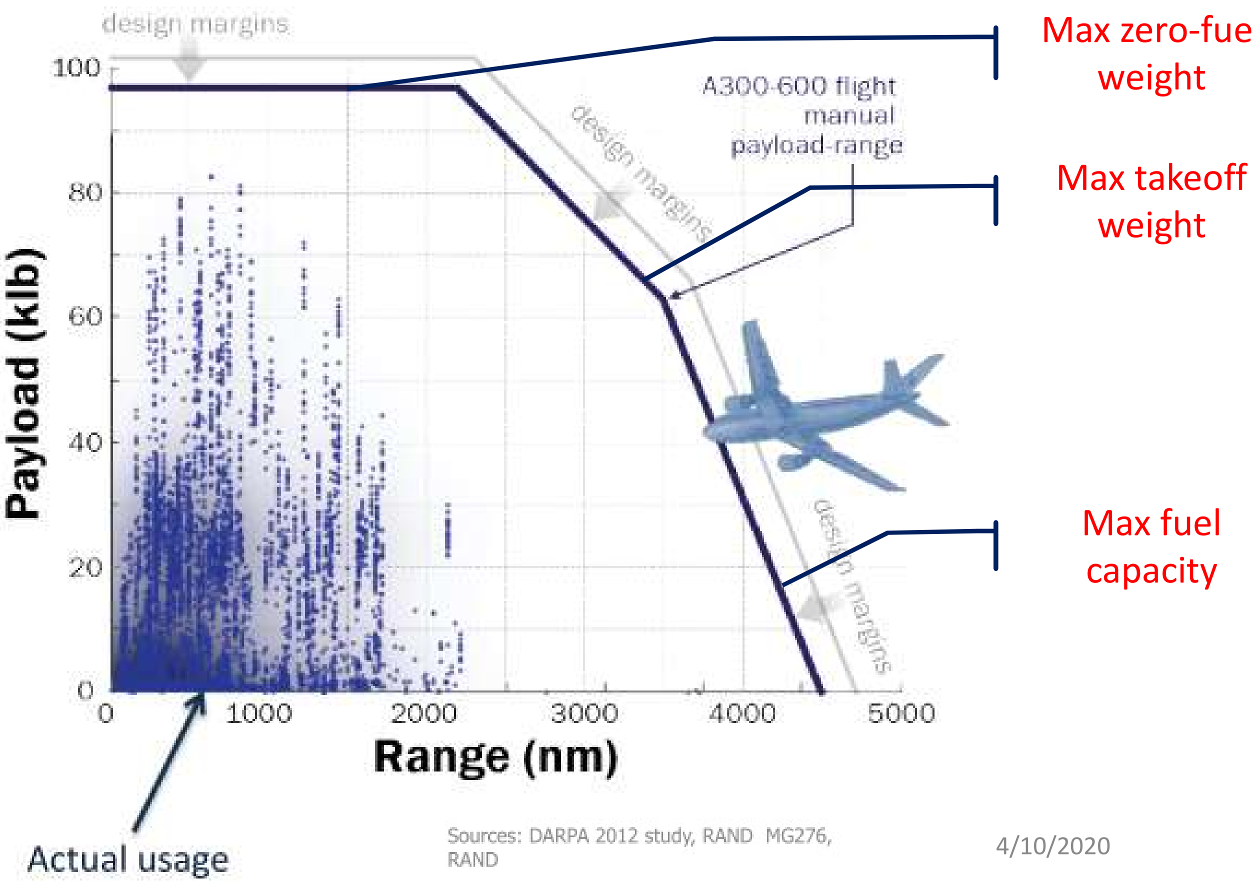
What to expect

- A method that:
  - turns stochastic design optimization problems into deterministic ones.
  - solves sparse non-linear problems (1:1 variables/constraints ratio).
  - solves in <1s for a conceptual design problem with ~150 variables.
  - has sub-linear solution time with number of variables.
  - provides probabilistic guarantees of constraint satisfaction.
- Insights into:
  - weaknesses of legacy methods of design under uncertainty.
  - how to reduce design conservativeness when faced with uncertainty.

Motivation: understanding how uncertainty influences design decisions.



Sources: DARPA 2012 study, RAND MG276, RAND

How about:

- Technological capabilities?
- Manufacturing quality?
- Regulatory environment?

Legacy methods are failing to adequately capture the risk/performance tradeoff.

- Margins
- Multimission design
- Off-nominal ‘checking’

- Not always intuitive.
- No quantitative measures of reliability.
- Heavy reliance on experienced engineers.
- Too conservative!

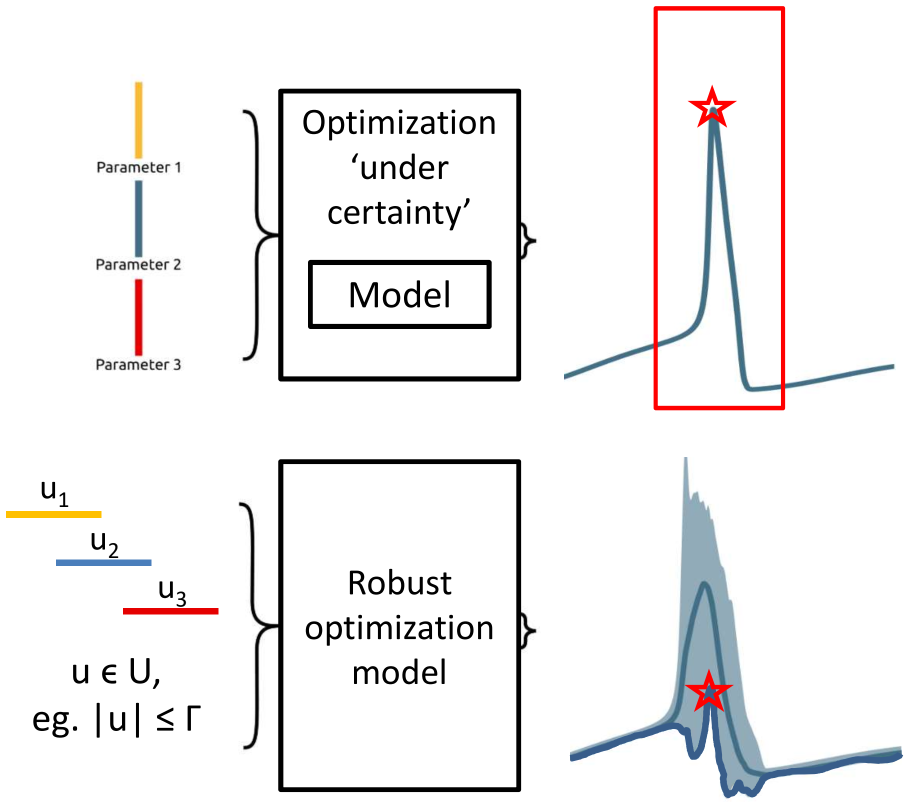
There is no such thing as
a free lunch!

We need more mathematical rigor in design under uncertainty.

- Guarantees of constraint satisfaction.
- Reduced sensitivity of designs to uncertain parameters.
- Better understanding of tradeoff of risk and performance.

Ultimate goal » *Less conservative* designs that are *robust*.

Robust optimization is a tractable method for OUU.



Objective space

*Adapted from: Tennøe, S., Halnes, G., and Einevoll, G. T., "Uncertainpy : A Python toolbox for uncertainty quantification and sensitivity analysis in computational neuroscience .," 2018, pp. 1–52.

The good:
- Monolithic and fast.
- Probabilistic guarantees.
- Tractable.

The bad:
- Doesn't make full use of distributional information.
- Optimizes worst case.

The (beautiful) and ugly:
- Requires specific formulations (LP, QCQP, SDP, GP, SP).

MATHEMATICAL BACKGROUND

GPs and SPs are practical to solve general NLPs.

Geometric program (GP):

- Log-convex
- Globally optimal
- No initial guesses
- Sensitivities through the dual

$$\begin{aligned} \text{minimize} \quad & p_0(\boldsymbol{x}) \\ \text{subject to} \quad & p_i(\boldsymbol{x}) \leq 1, \quad i = 1, ..., n_p, \\ & m_i(\boldsymbol{x}) = 1, \quad i = 1, ..., n_m, \end{aligned}$$

$$m(\boldsymbol{x}) = c \prod_{j=1}^{n} x_j^{a_j},$$

eg. $L = \frac{1}{2}\rho V^2 C_L S$

$$p(\boldsymbol{x}) = \sum_{k=1}^{K} c_k \prod_{j=1}^{n} x_j^{a_{jk}},$$

$P_t = P + \frac{1}{2}\rho V^2$

Signomial program (SP):

- Non-log-convex (difference of convex), and thus more general
- Solved as sequential GPs
- Solves with initial vector of 1's
- Locally optimal

$$\begin{aligned} &\text{minimize } f_0(\mathbf{x}) \\ &\text{subject to } f_i(\mathbf{x}) - h_i(\mathbf{x}) \leq 0, i = 1, ..., m \end{aligned}$$

Formulated in: GPkit

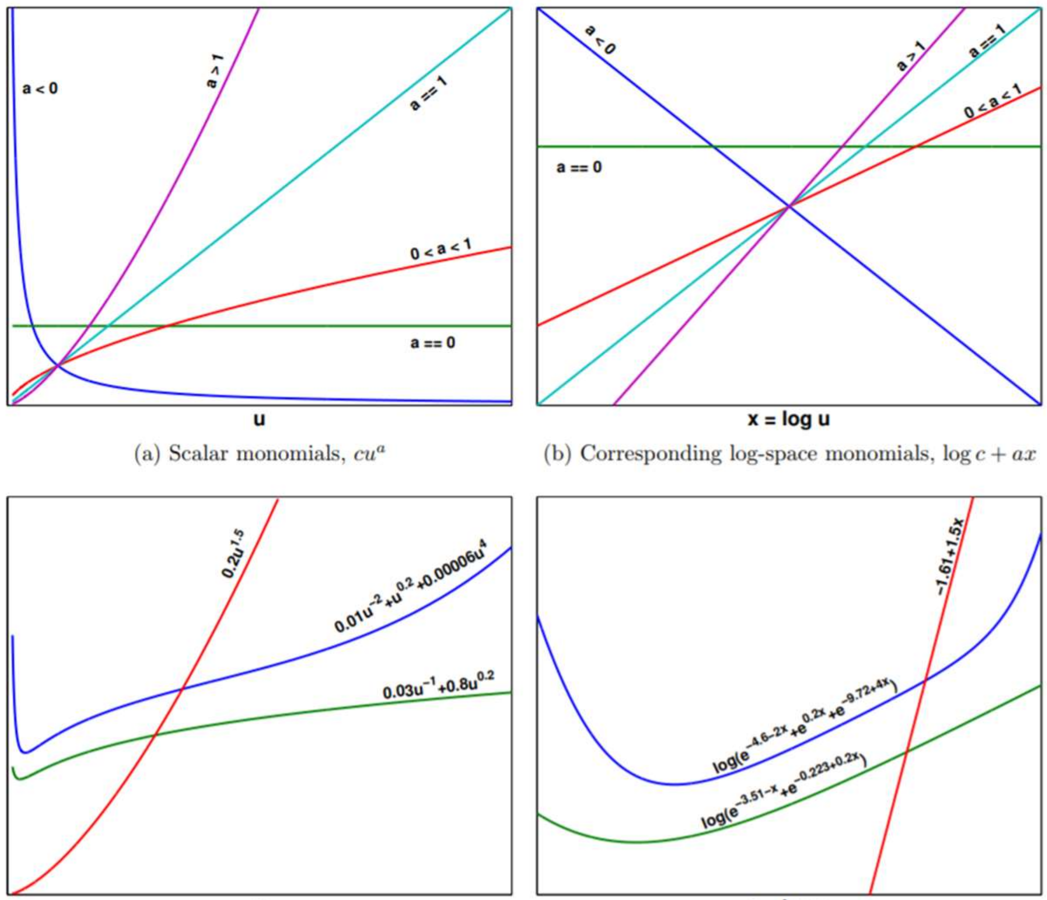
Log-log transformation to turn NLP into convex problem

Primal form monomial

Exponential form monomial

Primal form posynomial

Exponential form posynomial



(a) Scalar monomials, cu^a

(b) Corresponding log-space monomials, $\log c + ax$

(c) Scalar posynomials, $\sum_k c_k u^{a_k}$

(d) Corresponding log-space posynomials, $\log \sum_k e^{\log c_k + a_k x}$

*Hoburg, 2013. *Aircraft Design Optimization as a Geometric Program*

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The robust counterpart transforms OUU to deterministic optimization problem.

Optimization over:

$$\min f_0(x)$$
$$\text{s.t. } f_i(x,u) \le 0,\ \forall u \in \mathcal{U},\ i = 1,\dots,n$$

Infinite number of constraints

$$\min f_0(x)$$
$$\text{s.t. } \max_{u \in \mathcal{U}} f_i(x,u) \le 0,\ i = 1,\dots,n$$

Finite number of constraints

$$\min f_0(x)$$
$$\text{s.t. } \max_{u} f_i(x,u) \le 0,\ i = 1,\dots,n$$
$$\|u\| \le \Gamma$$

A well-defined set

Mathematical moves to obtain RSPs

- LPs have tractable robust counterparts.
- Two-term posynomials are LP-approximable.
- All posynomials are LP-approximable.
- GPs have robust formulations.
- RSPs can be represented as sequential RGPs.

LPs have tractable robust counterparts.

$$
\begin{aligned}
&\textit{minimize} && \boldsymbol{c}^\top \boldsymbol{x} \\
&\textit{subject to} && \boldsymbol{a}_i \boldsymbol{x} \le b_i, \quad \forall \boldsymbol{a}_i \in \mathcal{U}_i, \quad \forall i = 1, \dots, m,
\end{aligned}
$$

$$\mathcal{U} = \{(\boldsymbol{a}_1, \dots, \boldsymbol{a}_m) : \boldsymbol{a}_i = \boldsymbol{a}_i^0 + \Delta_i u_i,\ i = 1, \dots, m, \quad ||u||_2 \le \rho\},$$

Robust counterpart

I tip my hat to the editor!

$$
\begin{aligned}
&\textit{mininize} && \boldsymbol{c}^\top \boldsymbol{x} \\
&\textit{subject to} && \boldsymbol{a}_i^0 \boldsymbol{x} \le b_i - \rho ||\Delta_i \boldsymbol{x}||_2, \quad \forall i = 1, \dots, m.
\end{aligned}
$$

A tractable SOCP!

A. Ben-Tal and A. Nemirovski. Robust solutions of uncertain linear programs. *Operations Research Letters*, 25(1):1–13, 1999.

Takeaway: Given that an SP-compatible formulation exists, we can form a tractable robust SP!

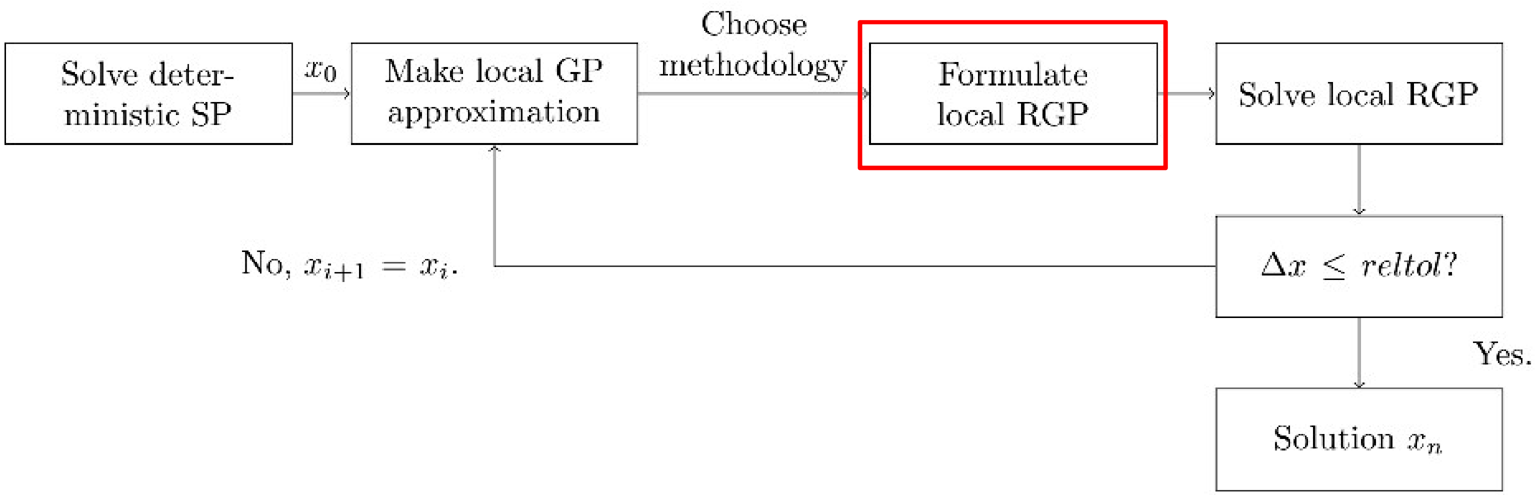
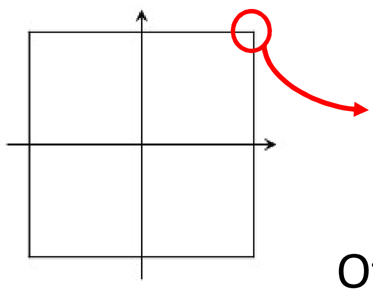


Figure 3: A block diagram showing the steps of solving an RSP.

Uncertainty sets considered

Box (L-∞ norm)

More conservative than margins.

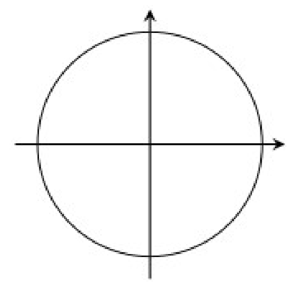


Margins optimize on a corner of the hypercube!

$p = \infty$

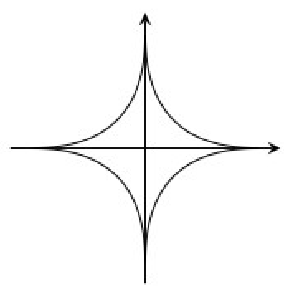
Elliptical (L-2 norm)

A less conservative candidate!

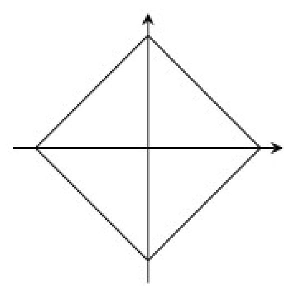


$p = 2$

Other norm sets also valid.



$p = \frac{1}{2}$



$p = 1$

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APPLYING RO TO CONCEPTUAL UAV PROBLEM

SP model captures important multidisciplinary tradeoffs.

- Unmanned, gas-powered aircraft
- Without uncertainty: 176 variables and 154 constraints
- Monolithic: optimizes aircraft and flight trajectory concurrently through disciplined SP form

Wing

- Structure
- Fuel volume
- Profile drag
- Stall constraint

Fuselage

- Ellipsoidal
- Fuel and payload
- Profile drag

Engine

- Data-based sizing
- Lapse rate
- BSFC fits
- T/O and TOC constraints

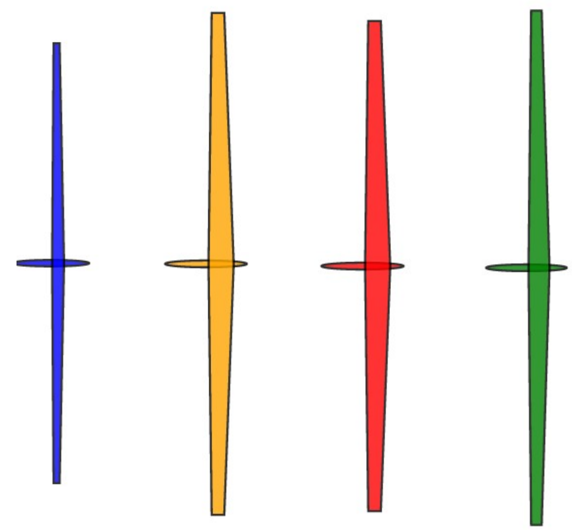
Uncertainties reflect 'engineering intuition'.

Table 1: Parameters and Uncertainties (increasing order)

Parameters	Description	Value	% Uncert. (3σ)
S_{wetratio}	wetted area ratio	2.075	3
e	span efficiency	0.92	3
μ	air viscosity (SL)	1.78×10^{-5} kg/(ms)	4
ρ	air density (SL)	1.23 kg/m^3	5
$C_{L_{\max}}$	stall lift coefficient	1.6	5
k	fuselage form factor	1.17	10
$C_{f_{\text{fuse,ref}}}$	fuselage skin friction factor	0.455	10
ρ_{p}	payload density	1.5 kg/m^3	10
τ	airfoil thickness ratio	0.12	10
N_{ult}	ultimate load factor	3.3	15
$V_{\min}$	takeoff speed	30 m/s	20
W_{p}	payload weight	6250 N	20
$W_{\text{w}_{\text{coeff,strc}}}$	wing structural weight coefficient	2×10^{-5} 1/m	20
$W_{\text{w}_{\text{coeff,surf}}}$	wing surface weight coefficient	60 N/m^2	20

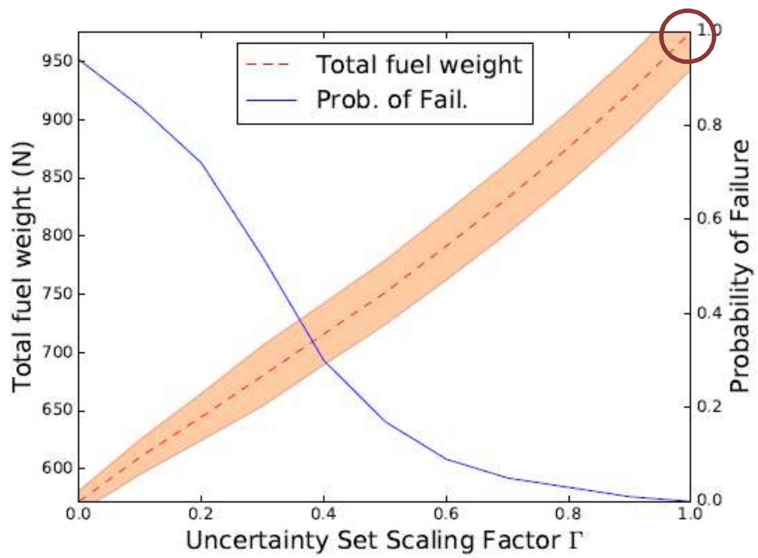
Table 2: SP Aircraft Optimization Results, for $\Gamma = 1$

Free variable	Description	Units	No Uncert.	Margins	Box	Elliptical
L/D	mean lift-to-drag ratio	-	33.6	23.6	25.1	27.7
AR	aspect ratio	-	24.6	13.3	13.0	16.3
Re	Reynolds number	-	1.54×10^6	2.65×10^6	3.03×10^6	250×10^6
S	wing planform area	m^2	13.6	32.8	32.0	28.1
V	mean flight velocity	m/s	41.6	37.3	38.9	38.4
T_{flight}	time of flight	hr	20.1	22.4	21.4	21.7
W_w	wing weight	N	2830	4760	4800	4480
$W_{w,strc}$	wing structural weight	N	2010	4760	2670	2620
$W_{w,surf}$	wing skin weight	N	820	2170	2120	1860
W_{fuse}	fuselage weight	N	250	314	288	279
$V_{f,avail}$	total fuel volume	m^3	0.0759	0.146	0.154	0.136
$V_{f,fuse}$	fuselage fuel volume	m^3	0.0394	0	0	0.0159
$V_{f,wing}$	wing fuel volume	m^3	0.0365	0.167	0.154	0.120
Objective metric	Description	Units	No Uncert.	Margins	Box	Elliptical
Objective	total fuel weight	N	608	1170	1240	1090
E[Objective]	expected total fuel weight	N	572	964	976	856
σ[Objective]	std. dev. of fuel weight	N	9	32	32	29
P[failure]	probability of failure	%	94	0	0	0

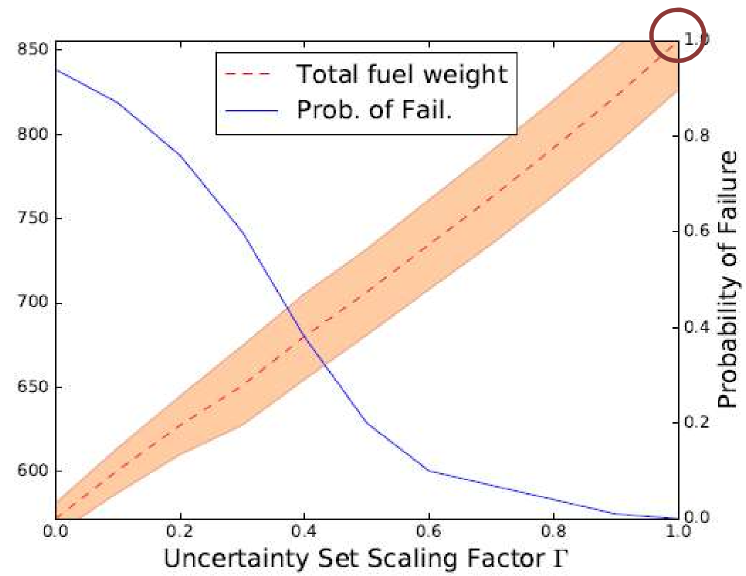


*100 MC simulations over 3σ truncated Gaussians

RSP successfully mitigates probability of failure.



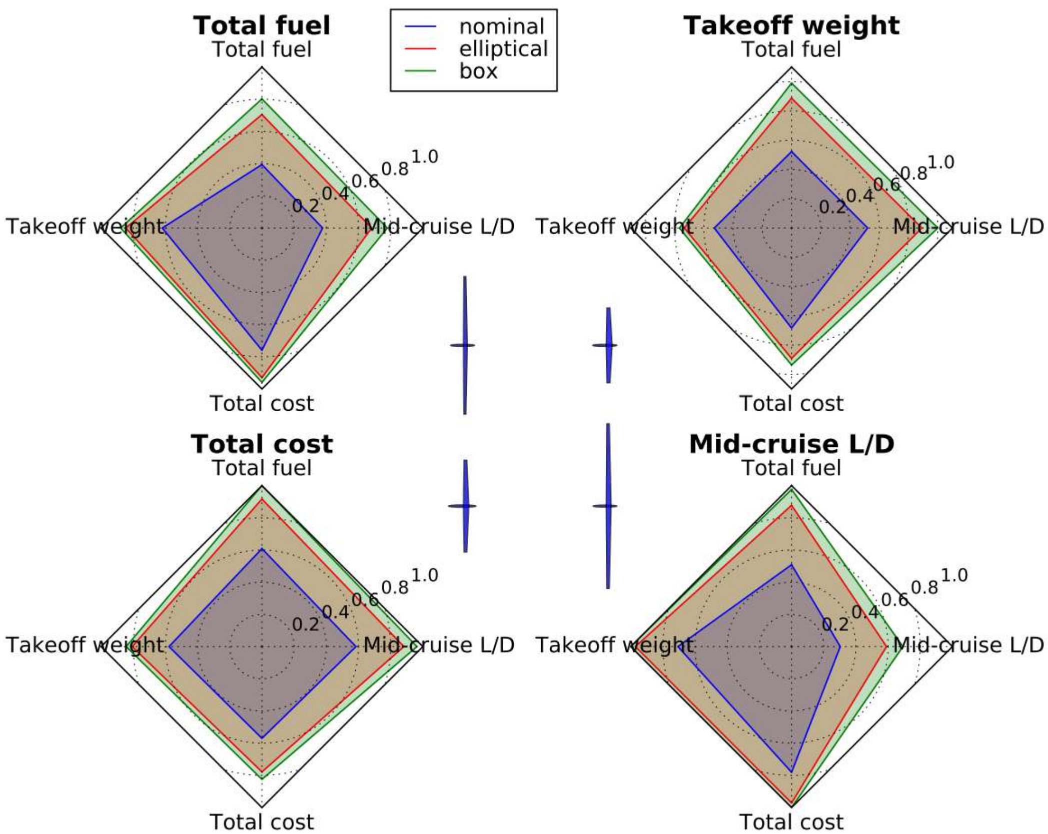
(a) Box Uncertainty Set

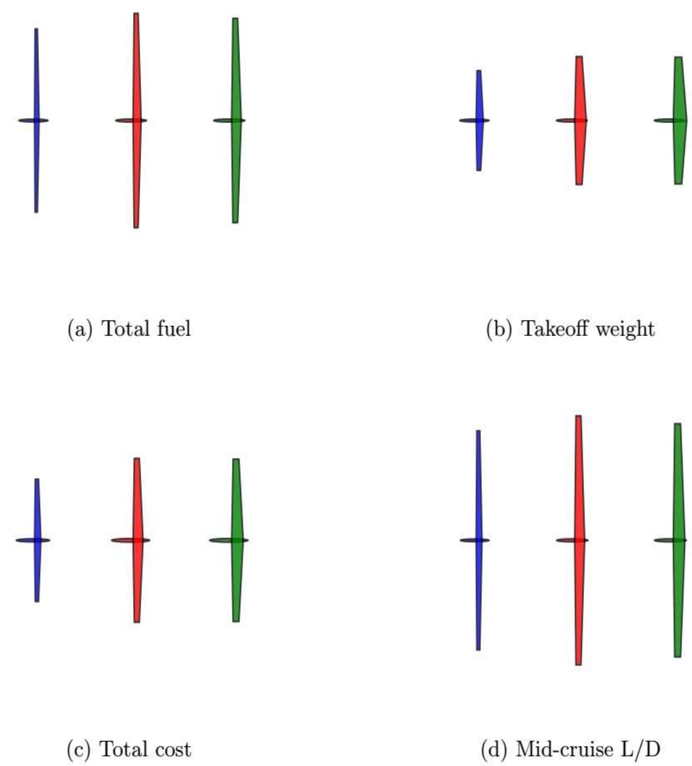


(b) Elliptical Uncertainty Set

For $\Gamma = 1$, the elliptical design spends 14% less fuel than the box design, while protecting against the same uncertainty!

Understanding multiobjective tradeoffs is key to risk mitigation.





Goal programming: risk is a global design objective.

Standard RO form

$$\begin{aligned} &\min\ f_0(x) \\ &\text{s.t.} \max_u\ f_i(x,u) \le 0,\ \ i = 1,\dots,n \\ &\qquad \|u\| \le \Gamma \end{aligned}$$

↔

Goal programming form

$$\begin{aligned} &\max\ \Gamma \\ &\text{s.t.} \max_u\ f_i(x,u) \le 0, i = 1,\dots,n \\ &\qquad \|u\| \le \Gamma \\ &\qquad f_0(x) \le (1+\delta) f_0^*,\ \ \delta \ge 0 \end{aligned}$$

RO form	Γ	δ	PoF	Goal form	δ	Γ	PoF
	0.00	2.5×10^{-4}	0.94		-	-	-
	0.10	0.057	0.87		0.057	0.10	0.87
	0.20	0.118	0.76		0.118	0.20	0.76
	0.30	0.183	0.60		0.183	0.30	0.60
	0.40	0.252	0.38		0.252	0.40	0.38
	0.50	0.326	0.20		0.326	0.50	0.21
	0.60	0.406	0.10		0.406	0.60	0.10
	0.70	0.492	0.07		0.492	0.70	0.07
	0.80	0.583	0.04		0.583	0.80	0.04
	0.90	0.681	0.01		0.681	0.90	0.01
	1.00	0.787	0.00		0.787	1.00	0.00

Suggests a good formulation for multi-objective design space exploration:

$$f_{0,j}(x) \le (1+\delta_j) f_{0,j}^*,\ \ \delta_j \ge 0,\ \ j = 1,\dots,m$$

Contributions

- A tractable RSP formulation for design over uncertain parameters
- Demonstration of
  - Probabilistic guarantees of RSPs
  - Less conservative designs through RSP than legacy methods
- A goal programming formulation for multiobjective optimization

Convex
Engineering

Please find our engineering design optimization packages and models at:
https://github.com/convexengineering

This work is powered by:
GPkit: .../gpkit
robust: .../robust (in development)
Mosek Version 8.1.0.80
Looking forward to your questions!

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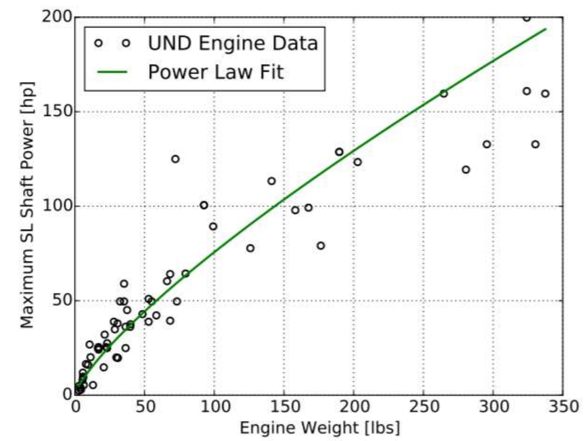
BACK-UP SLIDES

Examples of constraints

```
constraints += [(W_e/W_e_ref)**1.92 >= 0.00441 * (P_shaft_max/P_shaft_ref)**0.759
                      + 1.44 * (P_shaft_max/P_shaft_ref)**2.90]
```

```
constraints = [f == l/r/2,
               f <= 6,
               k >= 1 + 60/f**3 + f/400,
                3*(S/np.pi)**p >= 2*(l*2*r)**p + (2*r)**(2*p),
                V == 4./6.*np.pi*r**2*l,
```




```
# Thrust and fuel burn
W_f_s >= self.aircraftP.engineP['BSFC'] * self.aircraftP.engineP['P_{shaft}'] * t_s,
self.aircraftP.engineP['T'] * self.aircraftP['V'] >= self.aircraftP['D'] * self.aircraftP['V'] + Wavg * dhdt,
```

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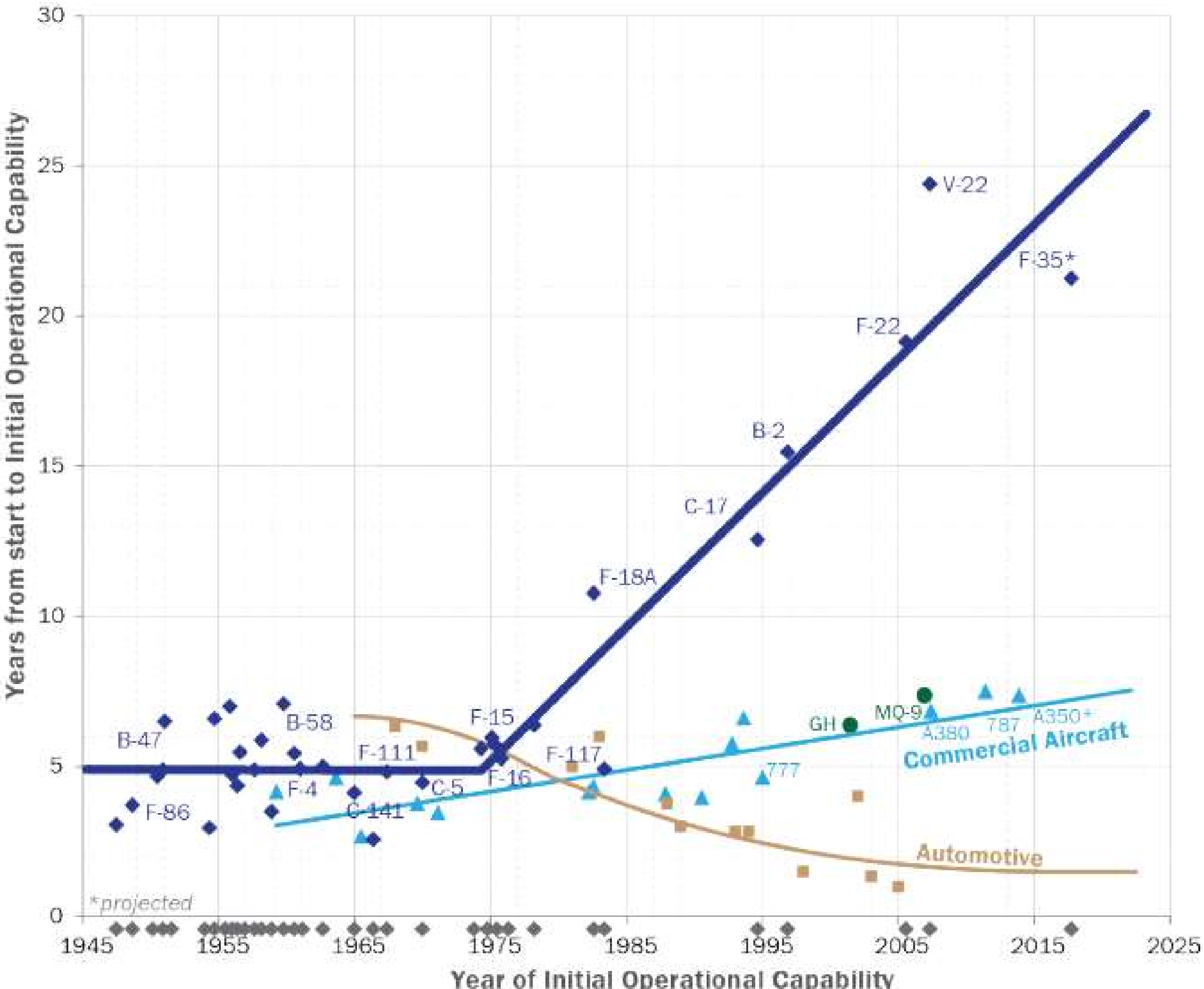
GPkit

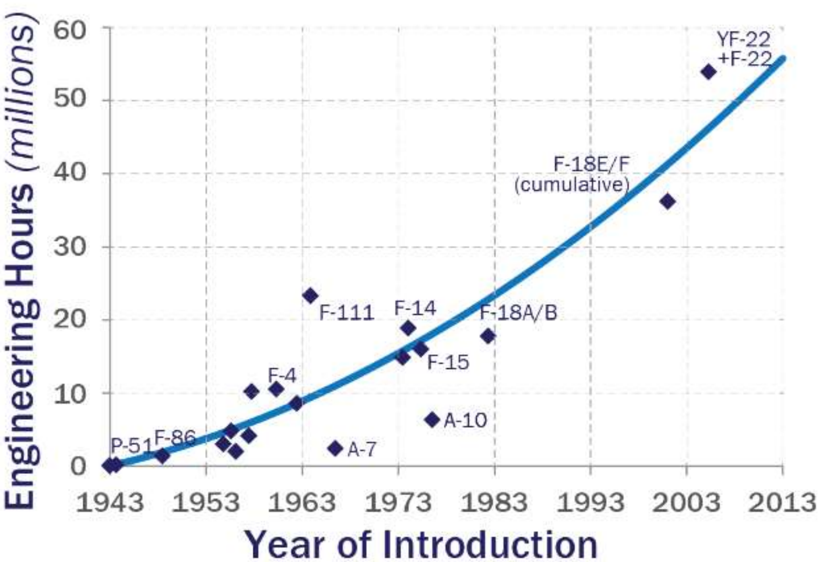


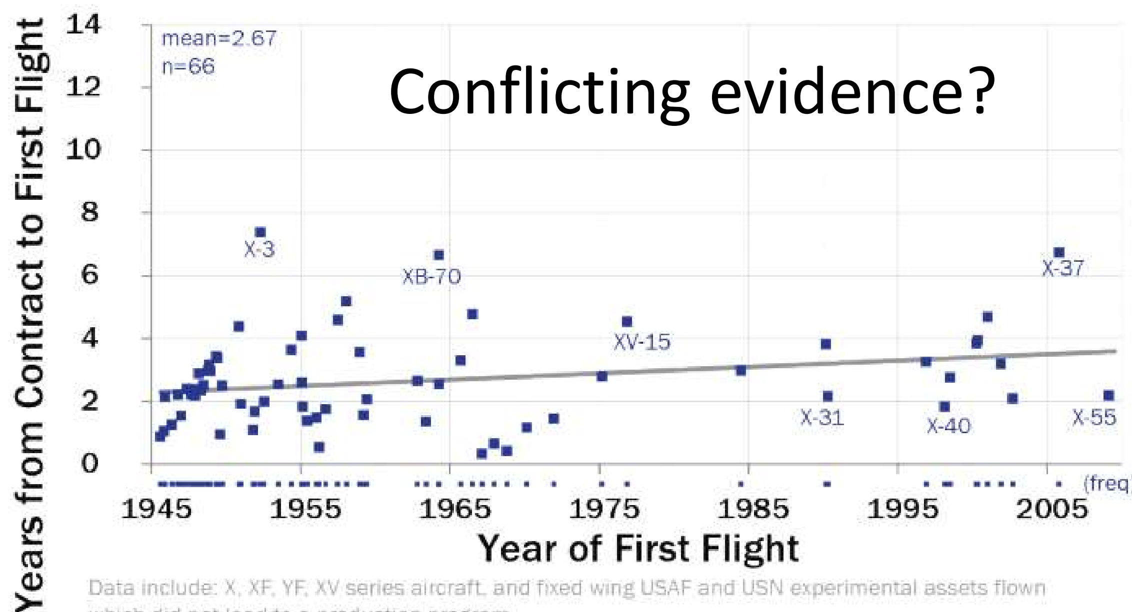
Distribution Statement "A" (Approved for Public Release, Distribution Unlimited)

How can we tackle the schedule and cost explosion of aerospace programs?

Sources: DARPA 2012 study, RAND MG276, RAND

We are approaching the limits of the 2nd law of thermodynamics.





Conflicting evidence?

Data include: X, XF, YF, XV series aircraft, and fixed wing USAF and USN experimental assets flown which did not lead to a production program.

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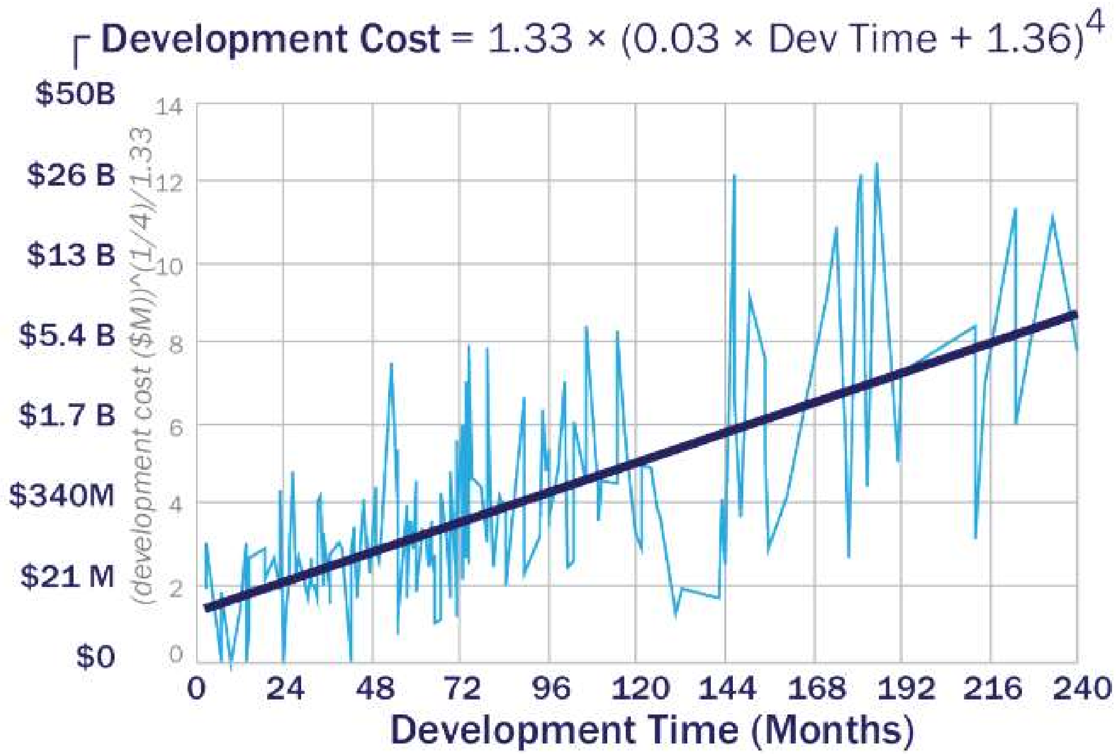
Sources: DARPA 2012 study, RAND MG276, RAND

Cost and schedule are highly correlated.



Source: McNutt, 1998. Survey of ~150 USAF ACAT I,II,III programs

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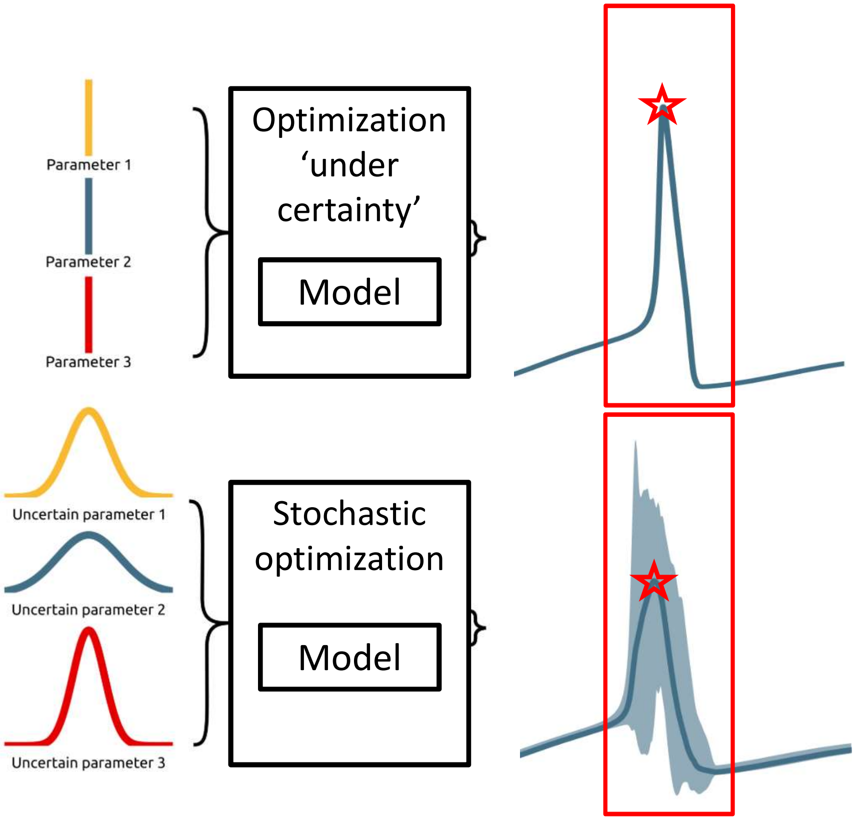
Stochastic optimization operates over PDFs.



Objective space

*Adapted from: Tennøe, S., Halnes, G., and Einevoll, G. T., "Uncertainpy : A Python toolbox for uncertainty quantification and sensitivity analysis in computational neuroscience .," 2018, pp. 1–52.

The good:

- Makes best use of available data.
- Extremely general.

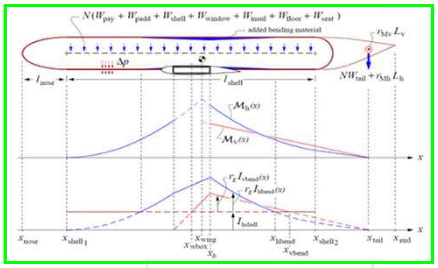
The bad:

- Not deterministic.
- 'Loose' probabilistic guarantees.

The ugly:

- Combinatorics/computational cost of PDF propagation through NLPs.

SPs can be extremely complex (TASOPT).



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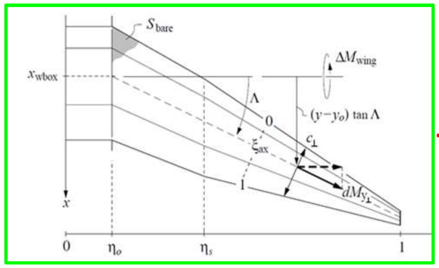

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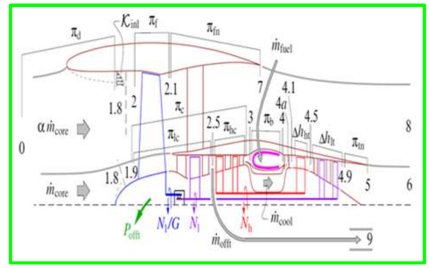

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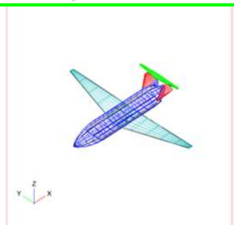

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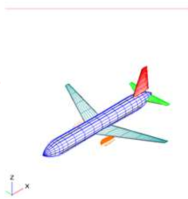


| Mission + sizing constraints]

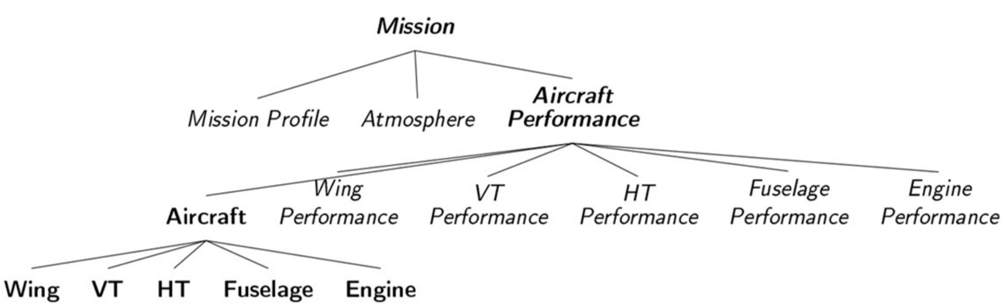
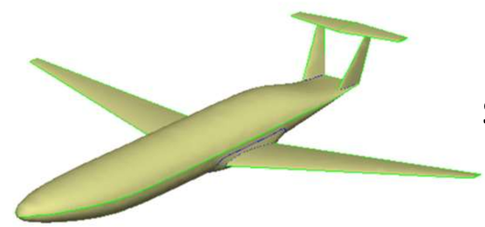


Figure 4-1: Hierarchy of the presented aircraft model. Models that include sizing variables are bolded while models that include performance variables are italicized. There are models that contain both kinds of variables.



- Commercial aircraft model of similar fidelity to TASOPT (5000 variables).
- Built on configuration hierarchies
- Multi-point design
- Visual debugging of constraints
- ESP integration for potential HF simulations?

Exponential form of GP

$$\begin{aligned} \text{minimize} \quad & f_0(\boldsymbol{u}) \\ \text{subject to} \quad & f_i(\boldsymbol{u}) \leq 1, i = 1, ..., m_p \\ & h_i(\boldsymbol{u}) = 1, i = 1, ..., m_e \end{aligned}$$

$$\begin{aligned} \min \quad & \textstyle\sum_{k=1}^{K_0} e^{\boldsymbol{a_{0k}}\boldsymbol{x}+b_{0k}} \\ \text{s.t.} \quad & \textstyle\sum_{k=1}^{K_i} e^{\boldsymbol{a_{ik}}\boldsymbol{x}+b_{ik}} \leq 1 \quad \forall i \in 1, ..., m \end{aligned}$$

$$h(\boldsymbol{u}) = e^b \textstyle\prod_{i=1}^{n} u_j{}^{a_j}$$

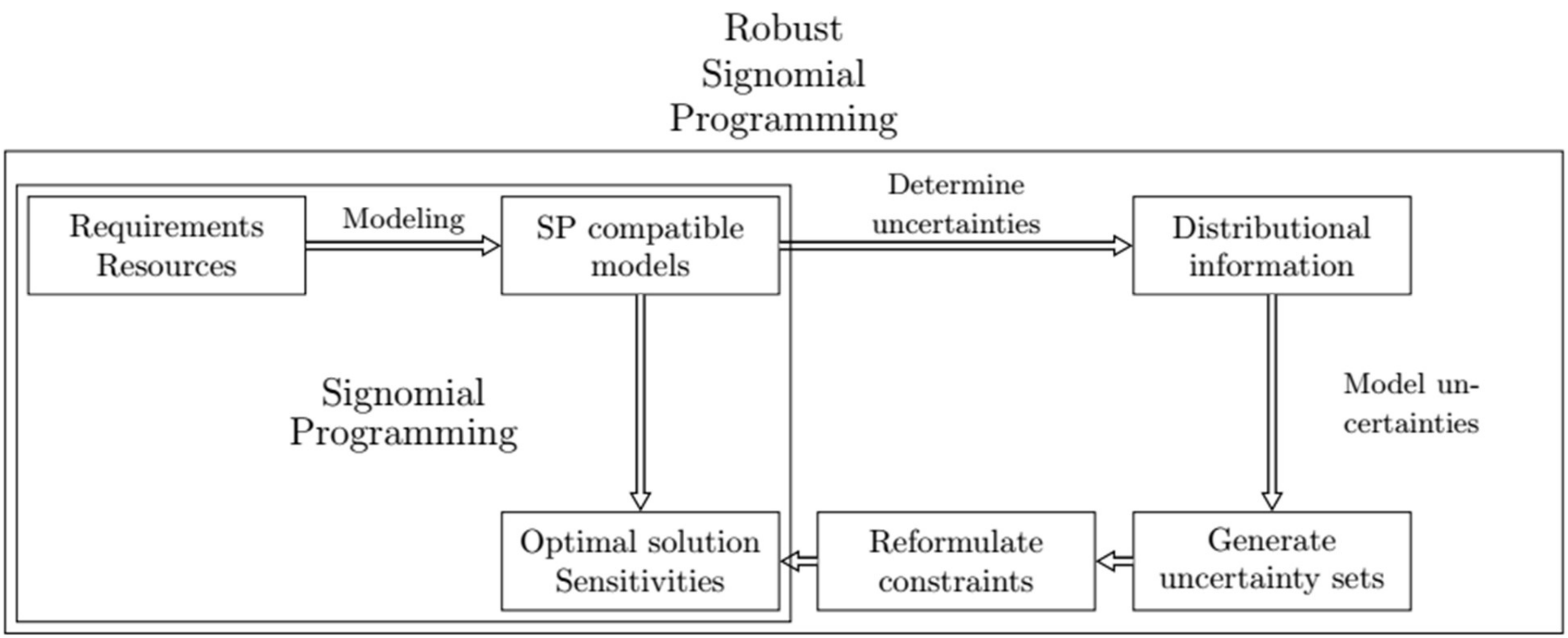
$$f(\boldsymbol{u}) = \textstyle\sum_{k=1}^{K} e^{b_{kj}} \prod_{j=1}^{n} u_j{}^{a_{kj}}$$

$$x_j = \log(u_j)$$

$$h_i(\boldsymbol{x}) = e^{\boldsymbol{a_i}\boldsymbol{x}+b_i}$$

$$f_i(\boldsymbol{x}) = \textstyle\sum_{k=1}^{K_i} e^{\boldsymbol{a_{ik}}\boldsymbol{x}+b_{ik}}$$

RSP formulations exist for all SP-compatible problems.



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Two-term posynomials are LP-approximable.

Corollary 1 *For $r \geq 3$, the unique best r-term PWL convex lower approximation $\underline{h}_r\colon \mathbf{R}^2 \to \mathbf{R}$ of the two-term log-sum-exp function is*

$$\underline{h}_r(y_1, y_2) = \max\{y_1, \underline{a}^\star_{r-2}y_1 + \underline{a}^\star_1 y_2 + \underline{b}^\star_1, \underline{a}^\star_{r-3}y_1 + \underline{a}^\star_2 y_2 + \underline{b}^\star_2, \ldots, \underline{a}^\star_1 y_1 + \underline{a}^\star_{r-2} y_2 + \underline{b}^\star_{r-2}, y_2\} \quad (24)$$

and the unique best r-term PWL convex upper approximation $\overline{h}_r : \mathbf{R}^2 \to \mathbf{R}$ is

$$\overline{h}_r(y_1, y_2) = \underline{h}_r(y_1, y_2) + \underline{\epsilon}_\phi(r), \quad (25)$$

where $a^\star_i, b^\star_i, i = 1, \ldots, r-2$ are the coefficients of the segments of $\underline{\phi}_r$ defined in (23).

Approximation error vs. degree of PWL approximation r.

Hsiung, K. L., Kim, S. J., and Boyd, S., "Tractable approximate robust geometric programming," *Optimization and Engineering*, vol. 9, 2008, pp. 95–118.

Given two-term posynomials are LP-approximable[1], all posynomials must then be LP-approximable.

The recipe:

$$
\begin{aligned}
\min \quad & f_0(\boldsymbol{x}) & & \\
\text{s.t.} \quad & \max_{\zeta \in \mathcal{Z}} \left\{ e^{\boldsymbol{a_{i1}}(\zeta)\boldsymbol{x} + b_{i1}(\zeta)} + e^{t_1} \right\} & \leq 1 & \quad \forall i : K_i \geq 3 \\
& \max_{\zeta \in \mathcal{Z}} \left\{ e^{\boldsymbol{a_{ik}}(\zeta)\boldsymbol{x} + b_{ik}(\zeta)} + e^{t_k} \right\} & \leq e^{t_{k-1}} & \quad \forall i : K_i \geq 4 \\
& & & \quad \forall k \in 2, ..., K_i - 2 \\
& \max_{\zeta \in \mathcal{Z}} \left\{ \sum_{k=K_i-1}^{K_i} e^{\boldsymbol{a_{ik}}(\zeta)\boldsymbol{x} + b_{ik}(\zeta)} \right\} & \leq e^{t_{K_i-2}} & \quad \forall i : K_i \geq 3 \\
& \max_{\zeta \in \mathcal{Z}} \left\{ \sum_{k=1}^{K_i} e^{\boldsymbol{a_{ik}}(\zeta)\boldsymbol{x} + b_{ik}(\zeta)} \right\} & \leq 1 & \quad \forall i : K_i \leq 2
\end{aligned}
$$

Simple example:

$$
\begin{aligned}
\min \quad & f & \\
\text{s.t.} \quad & \max\{M_1 + M_2 + M_3 + M_4\} & \leq 1 \\
& \max\{M_5 + M_6\} & \leq 1
\end{aligned}
$$

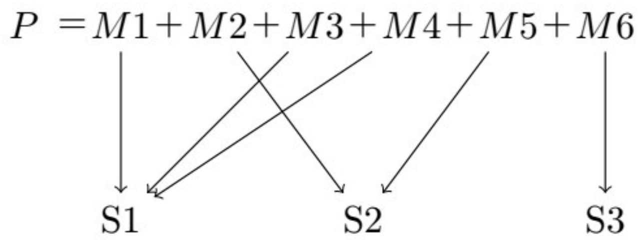
⇓

$$
\begin{aligned}
\min \quad & f & \\
\text{s.t.} \quad & \max\left\{M_1 + e^{t_1}\right\} & \leq 1 \\
& \max\left\{M_2 + e^{t_2}\right\} & \leq e^{t_1} \\
& \max\{M_3 + M_4\} & \leq e^{t_2} \\
& \max\{M_5 + M_6\} & \leq 1
\end{aligned}
$$

1. Hsiung, K. L., Kim, S. J., and Boyd, S., "Tractable approximate robust geometric programming," *Optimization and Engineering*, vol. 9, 2008, pp. 95–118.

Saab, A., Burnell, E., and Hoburg, W. W., "Robust Designs Via Geometric Programming." 2018. ArXiv:1808.07192

Uncoupled posynomials are robustified separately.



$$max\{P\} \leq 1 \iff \begin{array}{lr} t_1 + t_2 + t3 & \leq 1 \\ max\{S_1\} = max\{M_1 + M_3 + M_4\} & \leq t_1 \\ max\{S_2\} = max\{M_2 + M_5\} & \leq t_2 \\ max\{S_3\} = max\{M_6\} & \leq t_3 \end{array}$$

Figure 2: Partitioning of a large posynomial into smaller posynomials requires the addition of auxiliary variables. S_i are posynomials with independent sets of variables.

Three approximations exist for RGP.

Increasingly conservative

- Simple conservative
  - Maximizes each monomial term separately
- Linearized perturbations
  - Separates large posynomial into decoupled posynomials
  - Robustifies smaller posy's using RLO techniques

Uncertain coefficients only

- Best pairs
  - Separates large posynomial into decoupled posynomials
  - Finds least conservative combination of monomial pairs

Uncertain coefficients and exponents

Saab, A., Burnell, E., and Hoburg, W. W., "Robust Designs Via Geometric Programming." 2018. ArXiv:1808.07192

Convex programs allow flexibility in objectives.

Objective	Takeoff weight	Engine weight	Total cost	Wing loading	Total fuel	Time cost	Aspect ratio	Cruise L/D
Takeoff weight	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Engine weight	1.43	0.37	1.58	0.63	0.95	1.85	3.06	1.00
Total cost	1.09	2.26	0.83	1.00	1.17	0.69	1.38	1.12
Wing loading	40.32	73.87	15.25	0.11	45.32	2.58	0.60	46.46
Total fuel	1.17	0.49	1.11	1.00	0.75	1.26	2.89	0.72
Time cost	4.63	101.82	3.24	1.00	9.95	0.40	0.40	8.37
Aspect ratio	3.91	51.28	4.01	0.37	11.59	0.82	0.06	12.31
Cruise L/D	1.34	2.67	1.14	0.74	0.97	1.21	2.69	0.58

Table 3: Non-dimensionalized variations in objective values with respect to the aircraft optimized for different objectives. Objective values are normalized by the total fuel solution.

Future work

- How do we use our understanding of the risk of constraint violation?
  Not all constraint violation is equal!
- How does RO change our understanding of the benefits of adaptable designs?
  (eg. modular, morphing, adaptively manufactured designs and design families)
- How can we gather data about parameters to best reduce uncertainty in feasibility/performance of designs?

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